Modeling and Simulation of Physiological Systems

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Presentation Outline

- The role of computer modeling in bioengineering
- Bone Modeling
- Biological Soft Tissue
- Skeletal Muscles



The role of computer modeling in bioengineering

- Bioengineering is a broad field of scientific, biological, medical and engineering disciplines
- In Bioengineering living systems, processes and materials are investigated together with nonliving subjects, environment and materials, in order to advance fundamental knowledge and improve life
- Biomedical engineering represents a subset of disciplines whose main purpose is to develop, design and manufacture products that will improve human health



1950s and earlier

- > Artificial kidney
- > X-Ray
- Electrocardiogram
- Cardiac pacemaker
- > Cardiopulmonary bypass
- Antibiotic production technology
- > Defibrillator

1970s

- Computer assisted tomography (CT)
- Artificial hip and knee replacement
- **Balloon catheter** >
- Endoscopy >
- Biological plant/food engineering >
- > The cochlear implant and stimulators

1990s until today

- Genomic sequencing and micro-arrays
- Positron emission tomography
- Image-guided surgery





> Heart valve replacement

Ultrasound > Vascular grafts

1960s

Intraocular lens/Contact lens

Blood analysis and processing

Flow cytometry and cell sorting

- - Magnetic resonance imaging (MRI)
 - Laser surgery
 - Vascular stents
 - Recombinant therapeutics
 - Pulse oximeter >
 - Inner earcanal digital hearing aid

1980s

The American Institute for Medical and Biological Engineering 'Hall of Fame' gives a perspective on the most significant technological advancements in bioengineering in the twentieth century





A schematic view of an integrated bioengineering science. Adapted from the website of the Department of Biomedical Engineering at Boston University



American Institute for Medical and Biological Engineering (AIMBE) : http://www.aimbe.org

Introduction to bioengineering modeling

- From a biomechanics standpoint, a model represents a mathematical interpretation of the mechanical behavior of a material body or system
- This type of model is commonly called a mechanical model because it relies on physical laws or empirical relations which are relevant for the considered problem
- Bioengineering models play the same role as do mechanical models in general fundamental and applied sciences
- They are used in basic biomechanical research and laboratory investigations and for medical and industrial applications



Introduction to bioengineering modeling

- For relatively simple conditions, models can be formulated using analytical approaches
- However, in the analysis of more complex problems, numerical or computational methods must be employed
- These computational models usually require significant software development and extensive use of computers
- Today, complex computational models for a biological system or process assume the integration of fundamental disciplines (physics, biology, chemistry) with numerical methods, computer science and medicine



Example: Virtual vascular surgery on the grid



Virtual vascular surgery on the grid: from the MRI or CT scan recording of the patient vascular surgery region, to automatic generation of the computational model and analysis of results, yielding options for the surgical procedure.



Example: Nanoparticle delivery of therapeutic and imaging agents b)



a)

Blood vessel

Nanoparticle Neovascular endothelium Tumour stroma Permeation enhancer e.g. MMP9



Molecular motor molecule e.g. actin Actin filaments

C)



A vision of a future multistage nanodevice. A nanoparticle selectively binds to the cancer neovascular endothelium releasing multiple agents that enable the drug to pass through biological barriers and reach the targeted tumor cell.







http://www.woodgrovesec.moe.edu.sg/cos/o.x?c=/wbn/pagetree&func=view&rid=1155645

- The bone tissue represents the basic constituent of the skeleton and belongs to the group of supportive connective tissues
- Like others, this connective tissue has two components: cells and intercellular substance
- The specific chemical compound of the intercellular substance and structural organization of the fibrous components determine the rigidity as the main characteristic of bone tissue



- The intercellular substance of bones is made out of organic and nonorganic matter
- The organic compound consists of collagen (90–95%) and interfibrillar substance (ground substance)
- There are two types of bones:
 - Fibrous, nonlamellar, woven bone
 - Lamellar, mature bone.





Structure of a long bone (according to Remagen 1989)



Bone density

- Bone density can be defined as bone mass per total volume of bone including any holes
- The bone density calculated in this way represents the mean density of the apparent material specimen and is also known as the 'apparent density'
- The bone volume fraction V_v representing ratio of bone volume over total volume:

 $V_V = V_B / V_T$

• Where V_B and V_T are the bone volume and total volume, respectively



Bone density

- The bone tissue is homogenous with density p_{tissue}
- The relationship between apparent density and bone tissue density can be written as:

$$\rho_{apparent} = V_V \rho_{tissue}$$

Bone Mechanical Properties

• For general physiological loading conditions the bone material can be considered linear elastic



Bone Mechanical Properties

- The constitutive stress—strain relationship shows that bone material behaves in a manner similar to that of other engineering materials^[1]
- Stress-strain curves in tension and compression consist of an initial elastic region, which is nearly linear
- This region is followed by yielding and considerable, nonelastic, 'plastic' deformation before a failure
- Bone tissue that is loaded into this nonelastic region will not return to its original configuration after the load is removed



Bone Mechanical Properties

- Elastic modulus and strength of bone tissue are not constant. They are dependent on rate of deformation
- Experimental analysis revealed that the apparent density is important for elastic modulus estimation and the following relationship was proposed^[1]

$$E_{axial} = E_c \dot{e}^{0.06} \left(\frac{\rho}{\rho_c}\right)^3$$

• Where E_{axial} is the elastic modulus of bone of apparent density ρ , tested at strain rate of $\dot{e} \left[s^{-1} \right]$ and elastic modulus of bone with an apparent density of ρ_c tested at strain rate of 1.0 s^{-1}



Finite Element Modeling

• The bone structure is usually modeled by 3D finite elements in order to capture the bone geometry, and we here write the dynamic equation of motion for a 3D finite element:

 $\mathbf{M}^{n+1}\ddot{\mathbf{U}} + {}^{n}\mathbf{K}\mathbf{U} = {}^{n+1}\mathbf{F}^{ext}$

where **M** and ^{*n*}**K** are the element mass and stiffness matrices, ^{*n*+1} $\ddot{\mathbf{U}}$ and **U** are nodal acceleration and displacement vectors, and ^{*n*+1} \mathbf{F}^{ext} is the external nodal force which includes structural external forces and action from the surrounding elements



Finite Element Modeling

• The stiffness matrix ⁿK can be written as:

$$^{n}\mathbf{K} = \int_{V} \mathbf{B}^{T} \ ^{n}\mathbf{C}\mathbf{B}dV$$

where \mathbf{B} is the strain-displacement matrix and $^{n}\mathbf{C}$ is the constitutive matrix



Bone fracture

- A fracture means that the continuity of bone is disrupted
- Force transmission through the bone is no longer possible in any direction
- A fracture results in loss of the structural integrity of bone, and a loss of its weight capacity.
- A fractured bone becomes mechanically functionless



Fracture treatment

Compression plate





a)



Schauwecker, F., Osteosynthese Praxis, Thieme, Verlag, 1998





FE modeling of femur comminuted fracture

Fixation by the Neutralization Plate

- In the finite element model the 3D isoparametric finite elements are used for the bone tissue, neutralization plate and screws
- The axial force of 70 daN is applied at the bone top cross section
- Nodes lying at the bone bottom cross section are restrained in all directions. Also, it is considered that there is no slip between the screws and plate or bone tissue





Finite element model of femur comminuted fracture fixed by neutralization plate. b) Boundary condition and load



FE modeling of femur comminuted fracture

- The bone tissue is modeled using the material model defined by the relation
- The solution is obtained using the following data for the bone tissue:

$$E_c = 22.1 \cdot 10^3 MPa$$

 $\rho_c = 1.8 \cdot 10^{-3} g / mm^3$
 $\dot{e} = 0.1 s^{-1}$

$$\rho = 2.1 \cdot 10^{-3} \, g \, / \, mm^3$$

where it is assumed that the strain rate is the same within the tissue



Fixation by the Neutralization Plate

Bone Modeling

FE modeling of femur comminuted fracture

• For the neutralization plate and screw materials the stainless steel is used, with Young's modulus $E = 2.1 \cdot 10^5 MPa$

and Poisson's ratio $\nu = 0.3$





Neutralization plate; c) Bone tissue



Fixation by the Neutralization Plate



Distribution of the effective stress within the neutralization plate along the line AB



Kojić et al., "Computer modeling in bioengineering', 2008

FE modeling of femur comminuted fracture

Fixation by the intramedullary nail

- The femoral shaft fracture fixed by an interlocking nail is modeled
- In this particular case we assume comminuted type of fracture
- Also, it is taken that the intramedullary nail is locked by two screws proximally and two screws distally











Fixation by the intramedullary nail

Bone Modeling



Distribution of the effective stress within the intramedullary nail along the line AB



Kojić et al., "Computer modeling in bioengineering', 2008

Hip fracture

- Hip fractures represent certainly the most important orthopaedic-traumatologic problem
- The fixation by parallel screws and the by the dynamic hip implants are the two methods for internal fixation of intracapsular fractures of the femoral neck
- The stability of the osteosynthetic structure is different for these two solutions due to different biomechanical relations between the bone and fixator devices





Solutions for the hip fracture. a) Parallel screws; b) Dynamic hip implant



Ristic and Bogosavljevic, The femoral neck fracture: a biomechanical study of two internal fixation techniques, Medicus, 2004

Hip fracture: finite element models

Fracture fixation by parallel screws

- The structure is loaded by two forces F_A and F_R
- The force is generated by the gluteal muscles which connect the greater trochanter with pelvis
- The screws are taken to be of stainless steel with the material characteristics



Fracture fixation by parallel screws



anterior aspect, c) anterolateral aspect, d) anteromedial aspect



Ristic and Bogosavljevic, The femoral neck fracture: a biomechanical study of two internal fixation techniques, Medicus, 2004
Fracture fixation by parallel screws

Bone Modeling



Effective stress distribution in lateral cortex of the proximal femur for the intracapsular fracture of the femoral neck (solution by parallel screws)



Fracture fixation by parallel screws





Bone Modeling

Hip fracture: finite element models

Fracture fixation by dynamic hip device

 Using the same material properties and loads as in the previous example, we obtain the finite element solution for stresses and strains



Fracture fixation by parallel screws

Bone Modeling



a) b) c) d) Finite element model of the femoral neck fixation by dynamic hip device a) ematic view, b) anterior aspect, c) anterolateral aspect, d) anteromedial aspect



Bone Modeling



Effective stress distribution in lateral cortex of the proximal femur for the intracapsular fracture of the femoral neck (solution by dynamic hip device)



Fracture fixation by parallel screws





Distribution of the effective stress within the lateral cortex of the proximal femur along the line A-F







- Soft tissues display nonlinear mechanical response, i.e. nonlinear interdependence between the loadings and deformation within the range of the physiological working conditions
- These nonlinearities come from shape change and also from the nonlinear constitutive relationships
- Time dependent constitutive phenomena, such as, stress relaxation when the strains are held constant over time, or viscous effects under dynamic deformation



- The main constituents of these tissues are the extracellular fibrous proteins collagen and elastin
- Mechanical characteristics of the collagen and elastin are very different
- Collagen is a relatively inextensible protein and dominates in tendons and ligaments, as well as in bone and skin
- Individual collagen fibers break at around 2% strain



- Within tissue these fibers have as significant initial slack with no stiffness
- In practical applications the collagen fibers can be modeled by a nonlinear constitutive law expressed as:

$$\sigma = c_1 \ln \left[1 - \frac{\exp(e) - 1}{c_2} \right] + c_3 e$$

where $e = \lambda - 1$ is the strain and λ is the fiber stretch







Kowe et al. Analysis of elastic and surface tension effects in the lung alveolus using finite element methods, 1986

- Elastin is an extensible protein in connective tissues giving them the elastic mechanical behavior
- It consists of polypeptide chains which are elongated and sparsely cross-linked and can experience large strains
- It can be considered that the constitutive law of elastin bundles is linear even in the domain of large strains
- Young's modulus of an elastin fiber is of order $10^5 Pa$



• The basic experiments for the determination of tissue constitutive laws, or material models, are uniaxial and biaxial tests

Uniaxial Test and Uniaxial Model

- The uniaxial test is the basic mechanical test for biological tissue, as it is in general for other engineering material
- If a strip of a tissue, dissect from a membrane, is stretched quasi-statically



• The basic experiments for the determination of tissue constitutive laws, or material models, are uniaxial and biaxial tests

Uniaxial Test and Uniaxial Model

- The uniaxial test is the basic mechanical test for biological tissue, as it is in general for other engineering material
- If a strip of a tissue, dissect from a membrane, is stretched quasi-statically





a)

b)

Uniaxial constitutive law for tissue. **a)** Schematic of uniaxial loading of material element; **b)** Stress-strain relationship of a strip cut from alveolar tissue. The tissue has a small hysteresis when unloaded. The loading and uloading curves are not smooth because stress relaxations and stress recovery were allowed during the experiment



Biaxial Test and Biaxial Model

- The uniaxial constitutive law is not sufficient to describe mechanical behavior of biological membranes under general loading conditions
- If a membrane is loaded in two orthogonal directions, the stress-strain relationships for these directions are not the same as in case of uniaxial loading
- Characterization of the membrane mechanical response under stretching in both directions is obtained by performing biaxial tests: a membrane squared strip is stretched in two orthogonal directions





Biaxial stress-strain relationships. a) Schematic of biaxial test; b) Experimental procedure for biaxial testing of isotropic membrane: circular sample is fixed around the rim and subjected to pressure which produces biaxial stress-strain state (stresses and strains in all directions are the same in the central region);

MEDICAL TECHNOL

Hidebrandt et al. Stress–strain relations of tissue sheets undergoing uniform two-dimensional stretch, J. Appl. Physiol, 1969



c) Data for uniaxial and biaxial loading and fitted curves for cat mesentery. d) Test results and fitted curves for in-plane loading of bovine pericardium under several constant ratios of Green-Lagrange strain



Hysteretic Model

- It was found experimentally that the connective tissue has hysteretic behavior when subjected to cyclic loading, which is particularly significant when muscle cells are present
- The experimentally recorded dependence between the tensional force and the material strip length
- The constitutive law for the hysteretic tissue is represented by two relationships:

$$\sigma_{\ell} = \sigma_{\ell}(\lambda)$$
 and $\sigma_{u} = \sigma_{u}(\lambda)$





Hysteretic response of smooth muscle tissue. a) Experimental results on airway dog trachealis muscle strips b) Constitutive model for tissue histeresis expressed as the stress-stretch relationship



Sasaki and Hoppin, Hysteresis of contracted airway smooth muscle, J. Appl. Physiol: Respirat. Environ. Exercise Physiol 1979

Viscoelastic Models

- The connective tissue is modeled by a system of fibers within an elastic medium
- When the tissue deforms the force transfers among fibers and relative and relative sliding among fibers occur generating internal frictional force:

$$T(x,t) = \left[signv(x,t)\right] \mu p(x,t) + b_{w}v(x,t)$$

where T(x,t) is the force unit length, v(x,t) is the relative velocity at a point x along the fiber, p(x,t) is the compressive stress between the fibers; and μ and b_w are the Coulomb and viscous friction coefficients





Fiber-fiber kinetics model of connective tissue. **a)** Schematics of the model, from top-down: extensional force F transferred among fibers as forces F_1 and F_2 , enlarged contact between fibers, distribution of force within fibers, distribution of contact traction T;



Koji' cet al., A numerical stress calculation procedure for a fiber–fiber kinetics model with Coulomb and viscous friction of connective tissue, Comp. Mech., 2003



b) Experimental stress-strain hysteretic loops when the tissue is loaded cyclically (according to Mijailovic 1991); **c)** Computed hysteresis (according to Mijailovic et al. 1993, 1994; Kojic et al. 1998, 2003)



Models of Surfactant Covering Tissue

- Biological membranes within organs are covered by surfactant
- The surfactant plays important role not only in biophysical processes, but even in biomechanical response of membranes due to action of surface tension of surfactant
- This is true in particular for lung microstructure
- The description of mechanical behavior of surfactant refers first of all to lung surfactant



Models of Surfactant Covering Tissue

- Surface tension depends on the surfactant area and has a hysteretic characteristic
- This hysteresis plays an important role in lung functioning and gas exchange deep in the lung
- In the mathematical description of surface tension γ we will use the relation $\gamma = \gamma (A / A_0)$

where A / A_0 is the ratio of the current area and the initial area of the surfactant (at a given point of the surfactant surface)



Modeling methods for isotropic tissue

- Mechanical models for soft tissue represent the nonlinear constitutive laws
- Displacements of soft tissue can in general be large, therefore the problems are geometrically nonlinear
- We here give a review of the basic finite element (FE) relations in the form used in tissue modeling
- These relations are also applicable to the element free Galerkin (EFG) method



Modeling methods for isotropic tissue

- In general physiological conditions, inertial forces can usually be neglected, therefore the tissue deformation can be considered as a quasi-static problem
- Solution is obtained by discretization of the deformable body using a discrete methods, such as FE or EFG methods
- The basic equation of balance of linear momentum for a finite element has the form:

$$\left(\sum_{k=1}^{n+1}\mathbf{K}_{L}+\sum_{k=1}^{n+1}\mathbf{K}_{NL}\right)_{tissue}^{(i-1)}\Delta\mathbf{U}^{(i)}=\sum_{k=1}^{n+1}\mathbf{F}_{tissue}^{ext}-\sum_{k=1}^{n+1}\mathbf{F}_{tissue}^{int(i-1)}$$

• which corresponds to the step 'n' and iteration 'i' in an incremental-iterative solution procedure



Modeling methods for isotropic tissue

$$\left({}^{n+1}\mathbf{K}_{L} + {}^{n+1}\mathbf{K}_{NL}\right)_{tissue}^{(i-1)} \Delta \mathbf{U}^{(i)} = {}^{n+1}\mathbf{F}^{ext} - {}^{n+1}\mathbf{F}^{int(i-1)}_{tissue}$$

- $\binom{n+1}{K_L}_{tissue}$ and $\binom{n+1}{K_{NL}} \equiv {}^{n+1}K_{NL}$ are the linear and geometrically nonlinear stiffness matrices
- ^{*n*+1}**F**^{*ext*} and ^{*n*+1}**F**^{int(*i*-1)}^{int(*i*-1)} are the external forces acting to the element, which include the action of the surrounding finite elements, and internal forces due to stresses within the tissue
- $\Delta \mathbf{U}^{(i)}$ is the vector of increments of nodal displacements; and the left upper index n+1 denotes end of the incremental step



Modeling methods for isotropic tissue



Urinary bladder discretized into shell finite elements. Base vectors \mathbf{g}_r and \mathbf{g}_s are tangent to the isoparametric r and \underline{s} lines. Unit vectors $\overline{\mathbf{p}}_1$ and $\overline{\mathbf{p}}_2$ of the principal stretches in the shell tangential plane, at a material point P; they are rotated for an angle a with respect to the local shell coordinate system $\overline{x}, \overline{y}$





Schematic representation of muscle macrostructure



- A single muscle fiber is a cylindrical, elongated cell. Each fiber is surrounded by a thin layer of connective tissue called endomysium
- Organizationally, thousands of muscle fibers are wrapped by a thin layer of connective tissue called the perimysium to form the muscle bundle
- Groups of muscle bundles that join into a tendon at each end are called muscle groups, or simply muscles
- The entire muscle is surrounded by a protective sheath called epimysium







Kojić et al., "Computer modeling in bioengineering', 2008

- Muscle contraction is said to be isometric when the muscle does not shorten during contraction, while it is isotonic when muscle does shorten and the tension on the muscle remains constant
- The velocity-load relationship shows that muscles shorten more slowly as the load is increased in isotonic contraction
- On the other hand, power output is maximized at moderate loads at which 40% to 45% of the free energy of ATP (adenosine triphosphate) hydrolysis is converted into mechanical work





Schematics of muscle FE modeling: from muscle as a deformable body to Hill's model. **a**) Muscle discretization into finite elements; **b**) A 3D finite element with integration points and muscle fiber; **c**) Elongation of muscle fiber under the stress $\sigma_{\xi\xi}$; **d**) Hill's threecomponent model



- Muscle deforms under external loading and internal excitation, and in general has large displacements and strains
- Muscle material has nonlinear constitutive relations
- Neglecting the inertial forces, we can form an incremental-iterative scheme for determining muscle motion
- Hence, we have the equilibrium equation of a finite element for a load step 'n' and iteration 'i'



$$\left({}^{n+1}\mathbf{K}_{L} + {}^{n+1}\mathbf{K}_{NL}\right)^{(i-1)} \Delta \mathbf{U}^{(i)} = {}^{n+1}\mathbf{F}^{ext} - {}^{n+1}\mathbf{F}^{int(i-1)}$$

where ${}^{n+1}\mathbf{K}_{L}^{(i-1)}$ and ${}^{n+1}\mathbf{K}_{NL}^{(i-1)}$ are the geometrically linear and geometrically nonlinear stiffness matrices for the end of step, $\Delta \mathbf{U}^{(i)}$ are the increments of nodal displacements

 $^{n+1}\mathbf{F}^{ext}$ and $^{n+1}\mathbf{F}^{int(i-1)}$ are the external and internal nodal forces

• The most important relation in muscle mechanics is Hill's equation. It refers to mechanical behavior of skeletal muscle in the tetanized condition


• This equation is given as: $(v+b)(S+a) = b(S_0+a)$

where S represents tension (tensional stress) in muscle, is the velocity of the contraction; a and b, and S_0 are constants

- The constant S_0 is the maximum tension that can be produced under isometric tetanic contraction
- The Hill equation can be rewritten in dimensionless form as:

 $\frac{S}{S_0} = \frac{1 - (v/v_0)}{1 + c(v/v_0)}, \text{ in which the maximum velocity is } v_0 = \frac{bS_0}{a},$ and the constant $c = \frac{S_0}{a}$





Tension-velocity curve corresponding to a muscle in the tetanized condition







Gordon AM et al, The variation in isometric tension with sarcomere length in vertebrate muscle fibers, J. Physiol., 1996



A simple model which reflects the mechanical behavior of muscle: Hill's functional model. CE is the contractile element, SEE is the series elastic element and PEE is the parallel elastic element; σ is the stress in the muscle fiber direction



Kojic M, et al., Modelling of muscle behaviour by the finite element method using Hill's three-element model, Int. J. Num. Meth. Engrg, 1998



Geometry of the contractile (CE) and nonlinear elastic serial (SEE) elements



Kojic M, et al., Modelling of muscle behaviour by the finite element method using Hill's three-element model, Int. J. Num. Meth. Engrg, 1998





A simple model which reflects the mechanical behavior of muscle: Hill's functional model. CE is the contractile element, SEE is the series elastic element and PEE is the parallel elastic element; σ is the stress in the muscle fiber direction



Kojic M, et al., Modelling of muscle behaviour by the finite element method using Hill's three-element model, Int. J. Num. Meth. Engrg, 1998

- Since the Hill three-component model is based on a single sarcomere, the single sarcomere model may not be adequate for composite muscle consisting of different fiber types
- To overcome this deficiency the multi-fiber was introduced
- The extended Hill's model consists of a number of series of contractile and serial elements, corresponding to various types of sarcomeres (active part of a muscle), coupled in parallel to the linear elastic element representing the connective tissue (passive part)



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