

# SIGNAL PROCESSING IN BIOMEDICAL ENGINEERING

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# 1. BIOMEDICAL SIGNALS

- Biomedical signal: a signal being obtained from a biologic system (human or animal)/originating from a physiologic process
- almost every part of the body produces electrical signals
- they contain useful diagnostic information, they are carriers of information, both useful and unwanted.

BIOSIGNAL	FREQUENCY
ECG	0,5Hz-100Hz
EEG	0,5hZ-75Hz
Arterial pressure wave	DC-40Hz
Body temperature	DC-1Hz
Respiration	DC-10Hz
Electromyograph	10Hz-5kHz
Nerve action potentials	10Hz-10kHz

The amplitudes of some common biomedical signals

# 1.SOURCES OF BIOMEDICAL SIGNALS

1. Bioelectric signals: generated by nerves cells and muscle cells.
  - Nerve and muscle, action potentials, electric field propagation, ECG, EEG, EMG GSR
2. Biochemical signals from living tissue or samples analyzed in a laboratory
  - $pO_2$ ,  $pCO_2$ , ion concentration, glucose levels
3. Biomechanical signals (often invasive measurements are needed)
  - Motion, tension, displacement, blood pressure, flow
4. Biomagnetic signals
  - Fields generated by brain, heart and lungs
5. Bio-acoustic signals
  - Heart sounds
  - Respiratory sounds
  - Joint and contraction of muscles sounds
6. Bio-optical signals
  - Fluoroscopic properties of amniotic fluid
  - Cardiac output measured by dye dilution technique

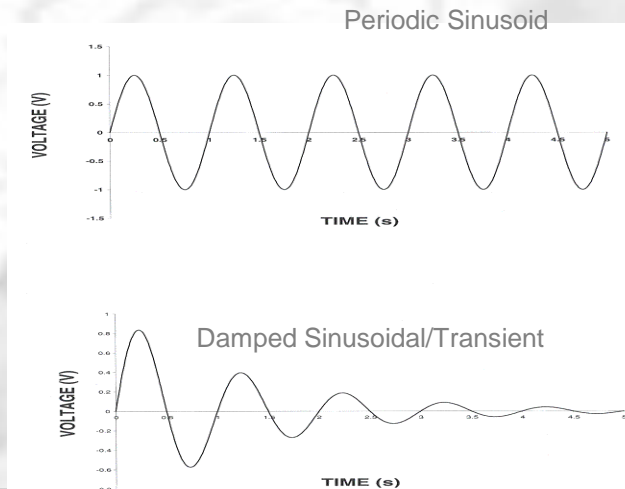
# 1. BIOMEDICAL SIGNAL CLASSIFICATION

- Signals can be classified as follows:
  - Continuous
    - A continuum of space or time
    - Continuous variable functions
  - Discrete
    - Discrete points in time or space
    - Represented as sequences of numbers
- Biomedical signals are almost always continuous

# 1. BIOMEDICAL SIGNAL CLASSIFICATION

Biomedical signals can be:

- Deterministic
  - Defined by mathematical functions or rules
    - Periodic signals are deterministic (sums of sinusoids)  $s(t)=s(t+nT)$
    - Transient signals can be deterministic: signal characteristics change with time
- Random
  - Are described by statistical or distribution properties
  - Stationary signals remain the same over time
    - Statistical
    - Frequency spectra



# 1. BIOMEDICAL SIGNAL CLASSIFICATION

- Real biomedical signals are not necessarily deterministic
  - Unpredictable noise
  - Non-stationary
    - Change in cardiac waveform over time
  - Identification of stationary segments of random signals is an important part of signal processing and pattern analysis
- Physiological and time domain signals can often be decomposed into a summation of sinusoidal component waveforms. *Fourier* analysis.
- The frequency and phase spectra contribute to the time domain behavior or shape of the signal.
- Modification of a signal in the frequency domain will affect the time domain behavior of the signal.

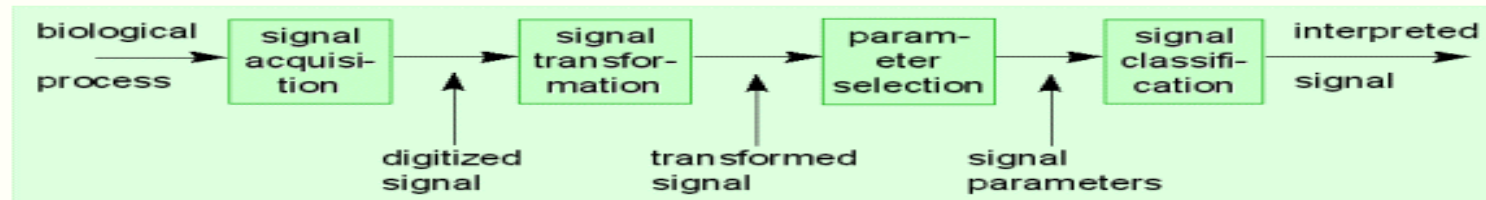
# 1. BIOMEDICAL SIGNAL PROCESSING

- Biomedical Signal Processing: the application of signal processing methods on biomedical signals
- involves the analysis of signals to provide useful information upon which clinicians can make decisions
- is an a 'operation' designed for extracting, enhancing, storing and transmitting useful information.
- is especially useful in the critical care setting, where patient data must be analyzed in real-time. Real-time monitoring can lead to better management of chronic diseases, earlier detection of adverse events such as heart attacks and strokes and earlier diagnosis of disease.



# 1. BIOMEDICAL SIGNAL PROCESSING

The four stages of biomedical signal processing



[1] Vinay K.Ingle,John G.Proakis, “ **DIGITAL SIGNAL PROCESSING Using MATLAB V.4**”, Northeastern University, 1997,

[10] B H Brown, R H Smallwood, D C Barber, P V Lawford and D R Hose, “ **MEDICAL PHYSICS AND BIOMEDICAL ENGINEERING**”

# 2.SHORT HISTORY

According to Alan V. Oppenheim and Ronald W. Schafer, the principles of signal processing can be found in the classical numerical analysis techniques of the 17th century. Oppenheim and Schafer further state that the "digitalization" or digital refinement of these techniques can be found in the digital control systems of the 1940s and 1950s

- Newton used finite-difference methods which are special cases of some discrete-time systems
- Gauss (1805) discovered the fundamental principle of the Fast Fourier Transform (FFT) even before the publication (1822) of Fourier's treatise on harmonic series representation of function (proposed in 1807)
- Early 1950s signal processing was done with analog system, implemented with electronics circuits or mechanical devices. First uses of digital computers in digital signal processing was in oil prospecting. The digital signal processing could not be done in real time.

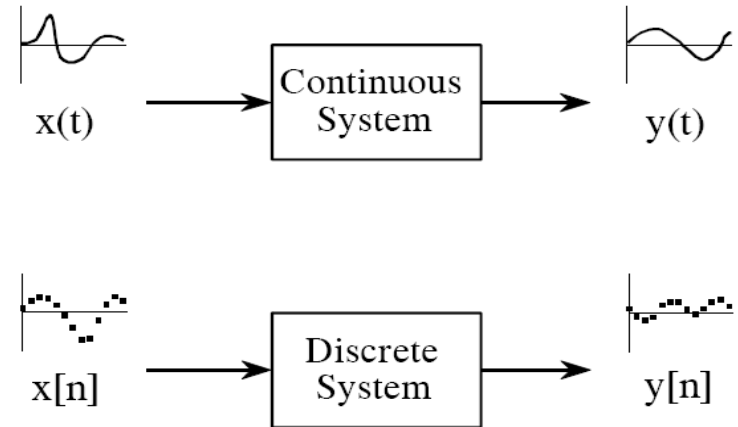
# 2.SHORT HISTORY

- FFT discovered by Cooley and Tukey in 1965
- The invention and proliferation of the microprocessor paved the way for low-cost implementations of discrete-time signal processing systems
- The mid-1980s, IC technology permitted the implementation of very fast fixed-point and floating-point microcomputer.
- The architectures of these microprocessor are specially designed for implementing discrete-time signal processing algorithm, named as Digital Signal Processors(DSP).

# 3.BASICS OF DSP

## Signals and LTI-Systems:

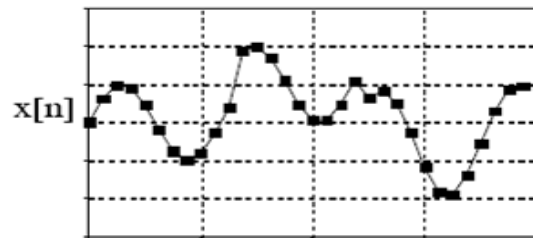
- Generation of an output signal in response to an input signal
- discrete and continuous systems



## Linear, Timeinvariant Systems:

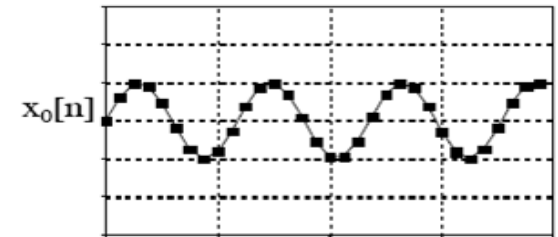
- additivity 
$$x[n] \xrightarrow{\text{System}} y[n] \Rightarrow kx[n] \xrightarrow{\text{System}} ky[n]$$
- homogeneity 
$$x_1[n] \xrightarrow{\text{System}} y_1[n], x_2[n] \xrightarrow{\text{System}} y_2[n] \Rightarrow x_1[n] + x_2[n] \xrightarrow{\text{System}} y_1[n] + y_2[n]$$
- shift invariance

# 3.BASICS OF DSP

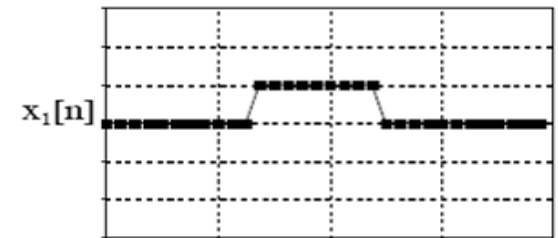


← synthesis

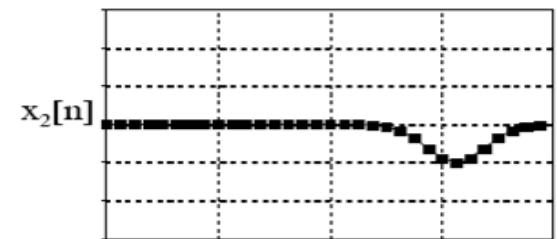
→ decomposition



+



+

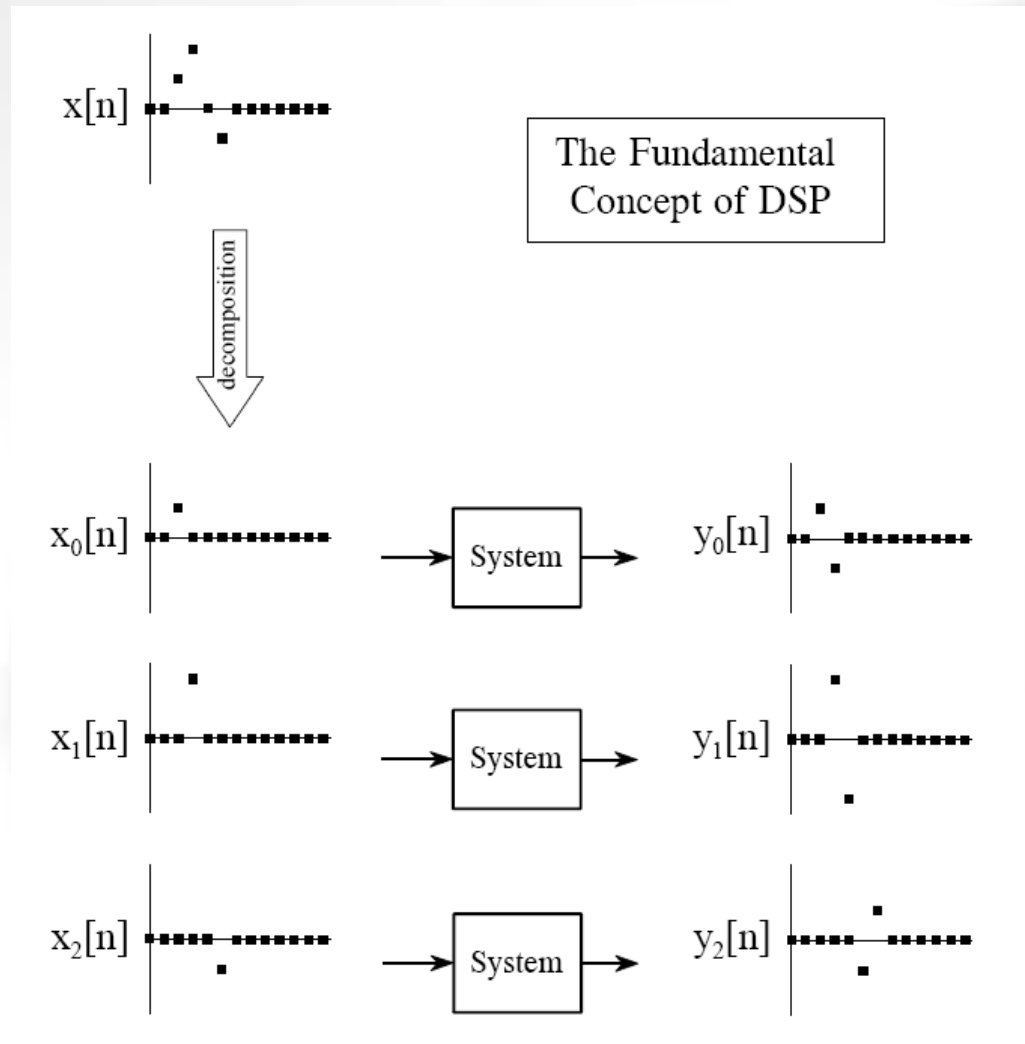


- **Synthesis:** in linear systems, signals can be combined by scaling and addition
- **Decomposition:** the inverse operation

[3] Alan V. Oppenheim, Ronald W. Schaffer, John R. Buck, "Discrete-Time Signal Processing", Prentice-Hall, Inc. 1999, 1989

[11] <http://www.dspguide.com/>

# 3.BASICS OF DSP-Superposition



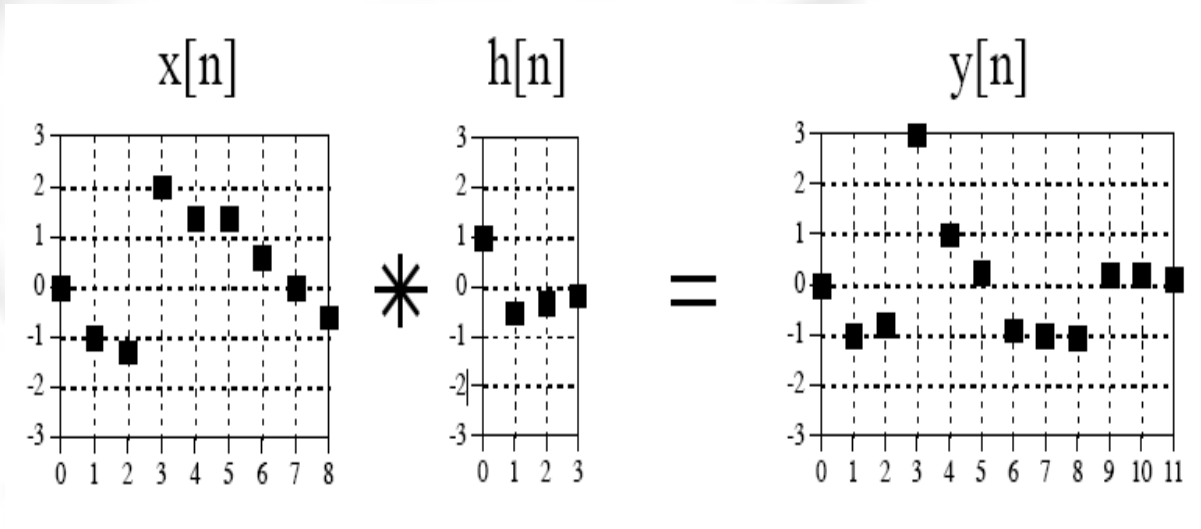
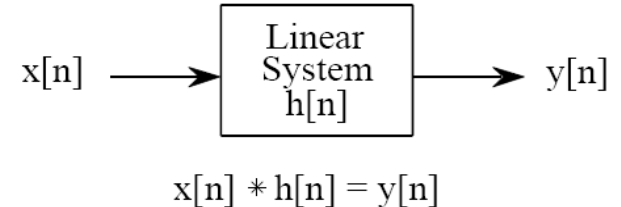
➤ Any signal can be **decomposed** into a group of additive components  $x_i$

➤ Passing these components through a linear system produces signals,  $y_i$

➤ The **synthesis** of these output signals produces the same signal as when  $x[n]$  is passed through the system

# 3.BASICS OF DSP-Convolution

- combined two signals into a third one
- applies a linear system to a signal via it's impulse response, which fully describes the system behaviour



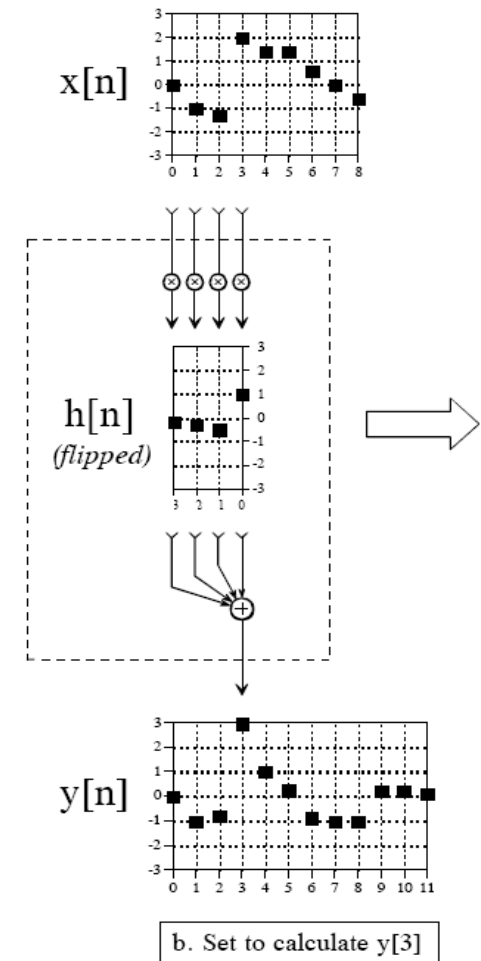
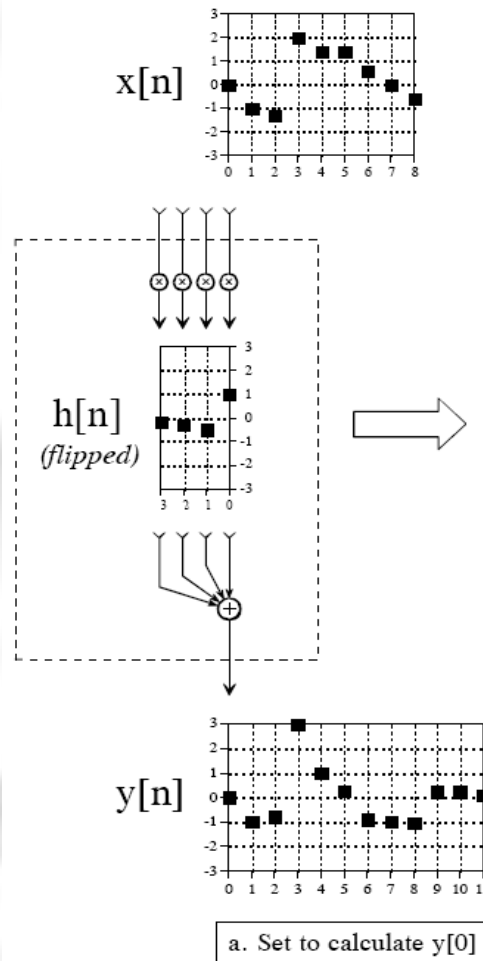
$$y[i] = x[i] * h[i] \iff y[i] = \sum_{j=0}^{M-1} h[j]x[i-j]$$

M .. length of  
impulse response

# 3.BASICS OF DSP-Convolution

Application of a LTI:

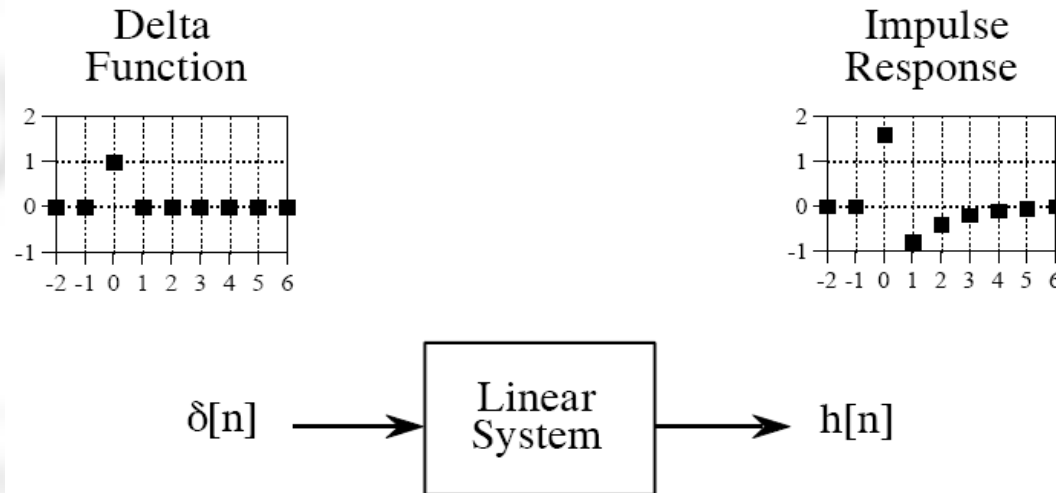
- multiplication of the input samples with the flipped impulse response
- addition of the values gives result for the corresponding output sample



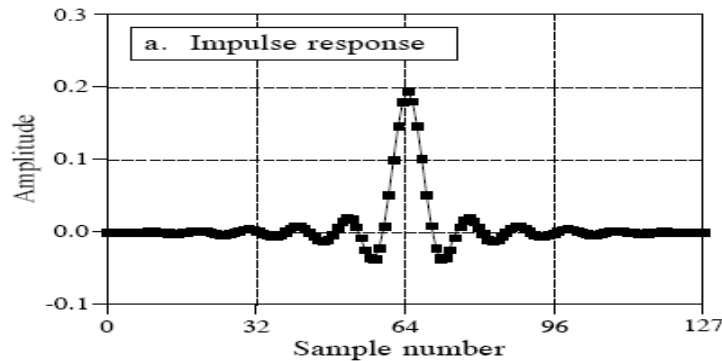


# 3.BASICS OF DSP-Convolution

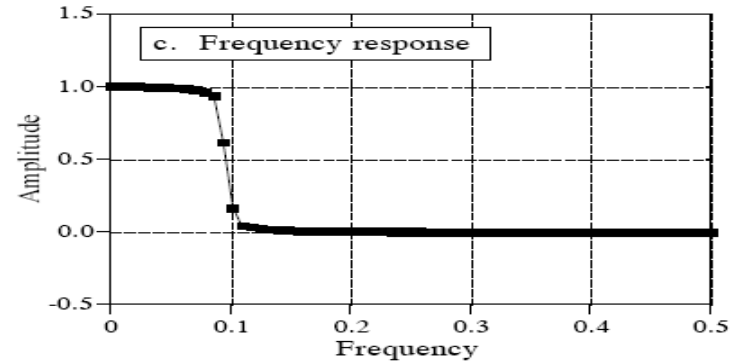
- many samples of the input signals contribute to one output sample
- the samples of the impulse response act as weighing coefficients
- feeding a delta function into a linear system gives the impulse response:



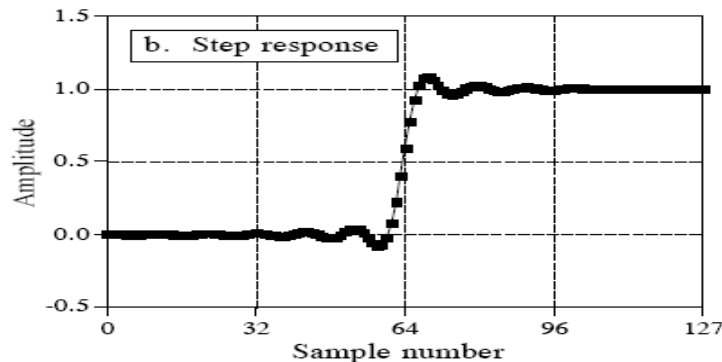
### 3.BASICS OF DSP-Relationships between impulse-, step- and frequency response:



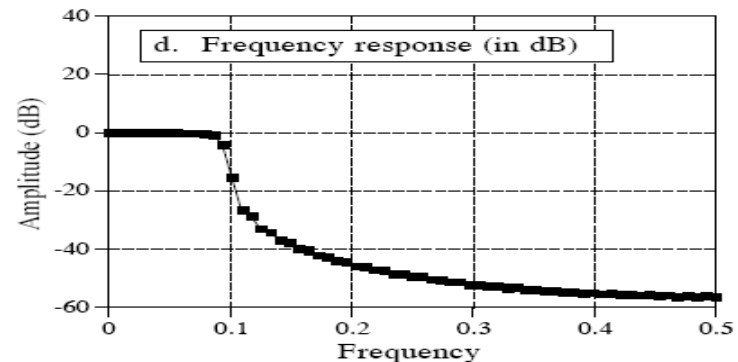
FFT  
→



↓ Integrate



↓ 20 Log( )



**Note: Convolution in time domain = multiplication in frequency domain !**

- [1] Vinay K.Ingle, John G.Proakis, "DIGITAL SIGNAL PROCESSING Using MATLAB V.4", Northeastern University, 1997,  
[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, "Discrete-Time Signal Processing", Prentice-Hall, Inc.1999,1989  
[11] <http://www.dspguide.com>

# 3.BASICS OF DSP-Convolution and FIR Filters

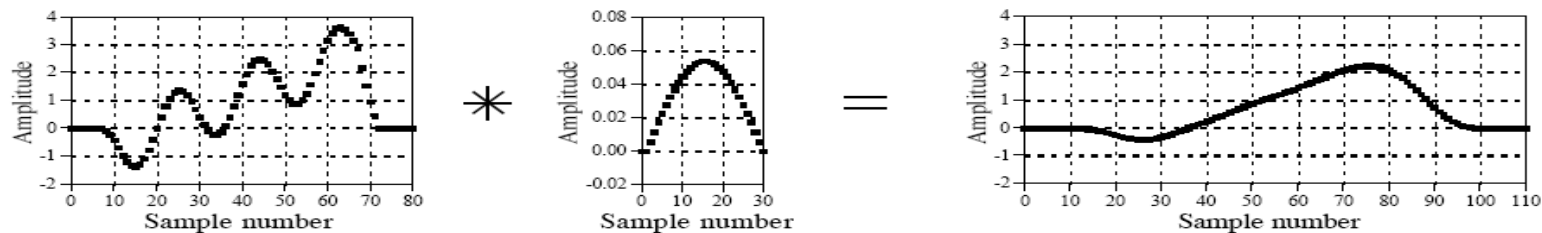
The shape of the impulse response determines phase- and frequency response of an LTI system. The impulse response is also called „filter kernel“.

- Finite Impulse Response Filters can be implemented by a single convolution of an input signal with the filter kernel
- Several positive values in the impulse response give an averaging (low-pass) filter
- Subtracting a low-pass filter kernel from the delta function gives a high pass filter kernel
- A symmetrical impulse response gives a linear phase response

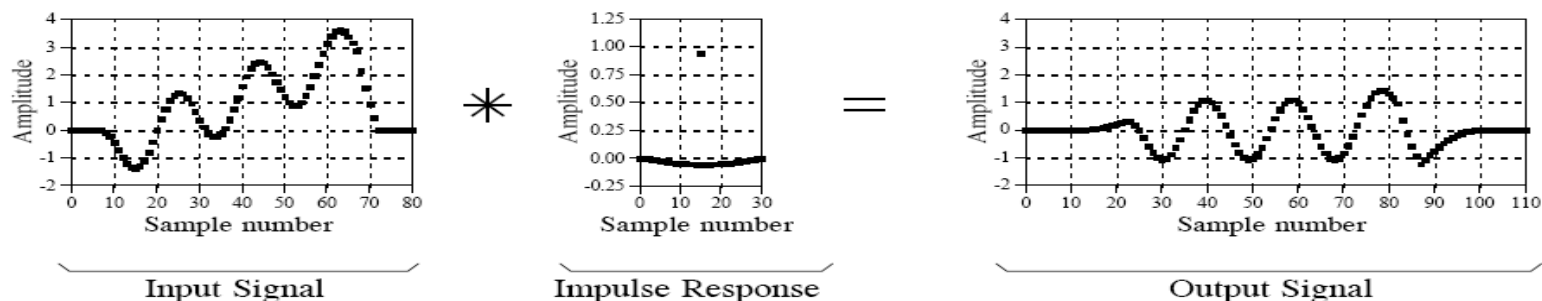
# 3.BASICS OF DSP-Convolution and FIR Filters

## ➤ Example High and Lowpass Filter-Kernels:

### a. Low-pass Filter



### b. High-pass Filter



# 3.BASICS OF DSP-Z-Transform

- Digital Filters can be described by the generalized discrete differential equation:

$$\sum_{m=0}^M \alpha_m \cdot y[n-m] = \sum_{k=0}^N b_k \cdot x[n-k]$$

a, b : filter coefficients

x[n] : input signal

y[n] : output signal

M,N :filter order

- ✓ the right side depends only on the inputs x[n] : feed-forward
- ✓ the left side depends on the previous outputs y[n] : feed-back

FIR Filters have only feed-forward components and they can be calculated non-recursively, by convolution

IIR Filters have feed-back components also and they are calculated recursively (infinite impulse response)

# 3.BASICS OF DSP-Z-Transform

- discrete version of the Laplace-transform
- using the Z-transform, the characteristics of a digital filter can be described by the following transfer function:  $x \longrightarrow \boxed{H(Z)} \longrightarrow y$

$$\sum_{m=0}^M a_m \cdot y[n-m] = \sum_{k=0}^N b_k \cdot x[n-k] \xrightarrow{Z} Y(z) \cdot \sum_{m=0}^M a_m \cdot z^{-m} = X(z) \cdot \sum_{k=0}^N b_k \cdot z^{-k}$$

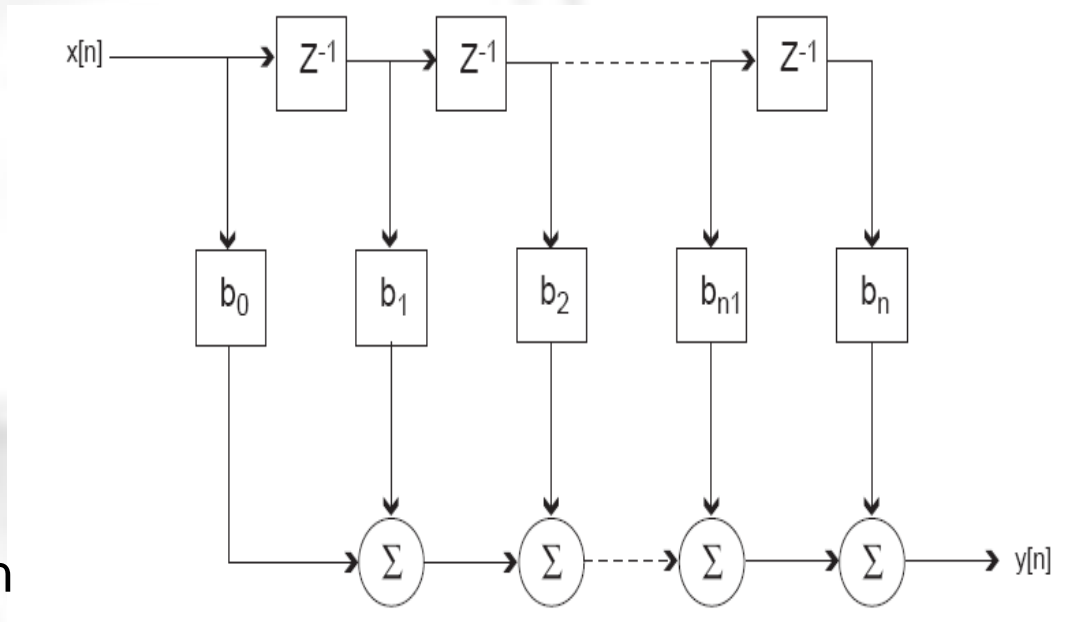
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^N b_k \cdot z^{-k}}{\sum_{m=0}^M a_m \cdot z^{-m}}$$

- ✓  $z^x$  in Z-domain represents a delay element of x discrete delays,
- ✓ the numerator describes the feedforward part of the filter, 0 = „zeros“
- ✓ the denominator describes the feedback part of the filter, 0 = „poles“

# 4. Digital Filters – FIR filters

$$y[n] = \sum_{k=0}^N b_k \cdot x[n-k]$$

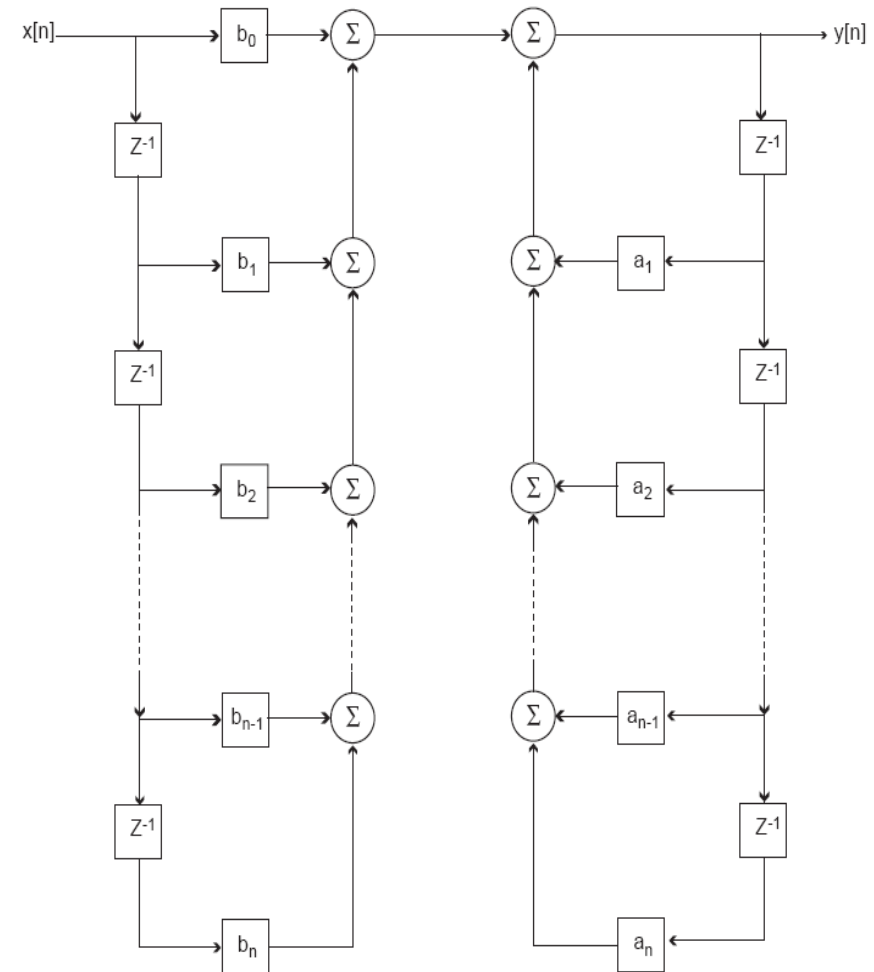
- finite impulse response, no recursion
- described by multiplication coefficients
- less sufficient (need higher order)



# 4. Digital Filters – IIR filters

$$y[n] = \sum_{k=0}^N b_k \cdot x[n-k] + \sum_{m=1}^M -a_m \cdot y[n-m]$$

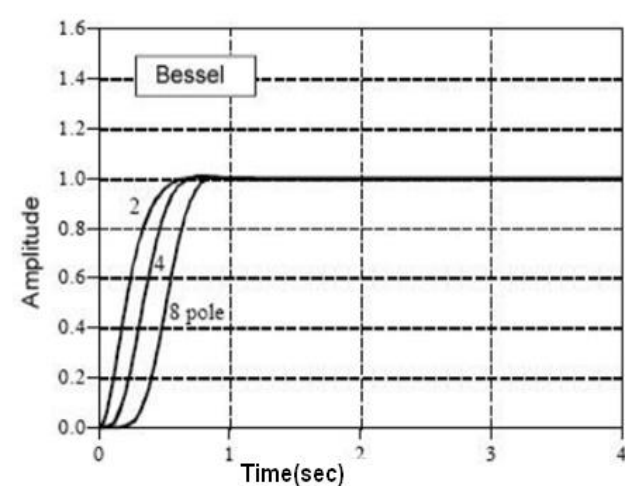
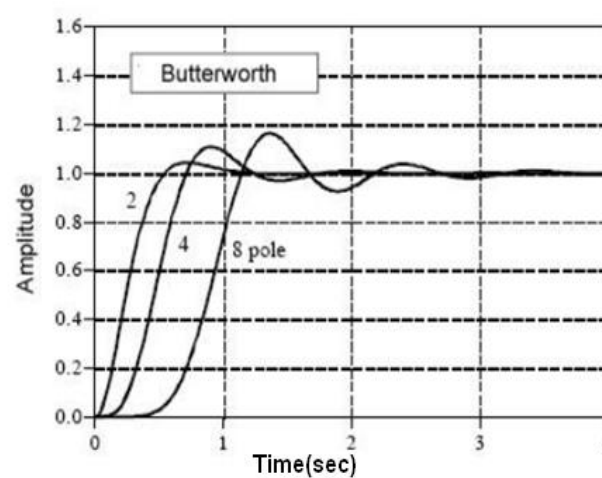
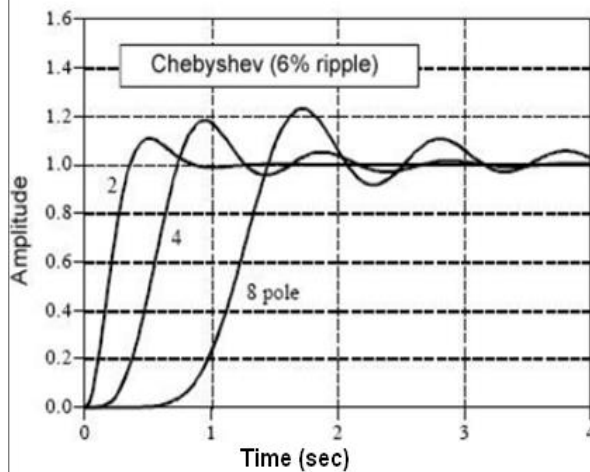
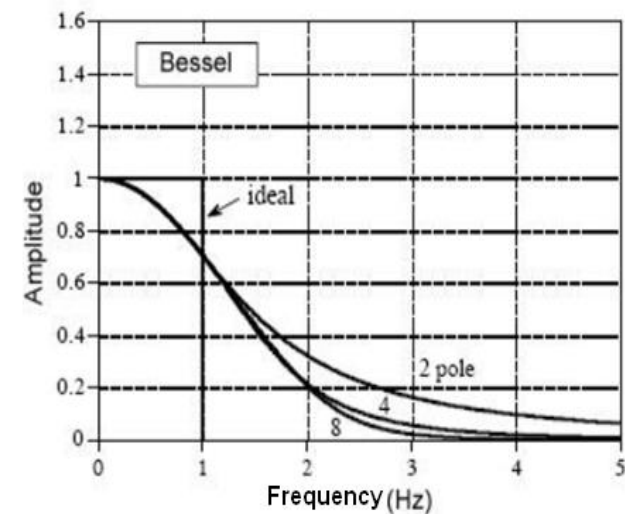
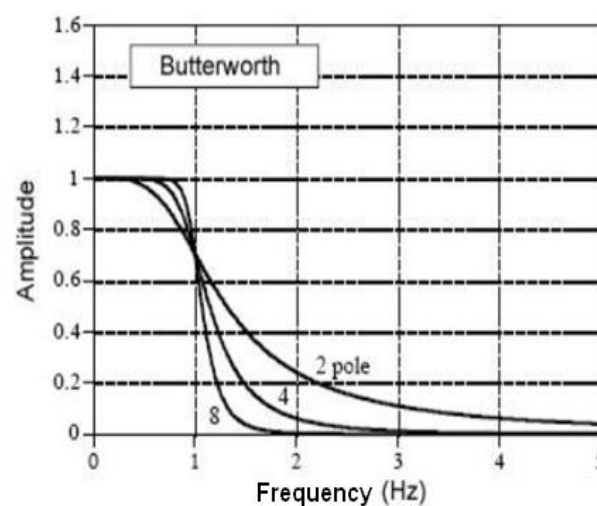
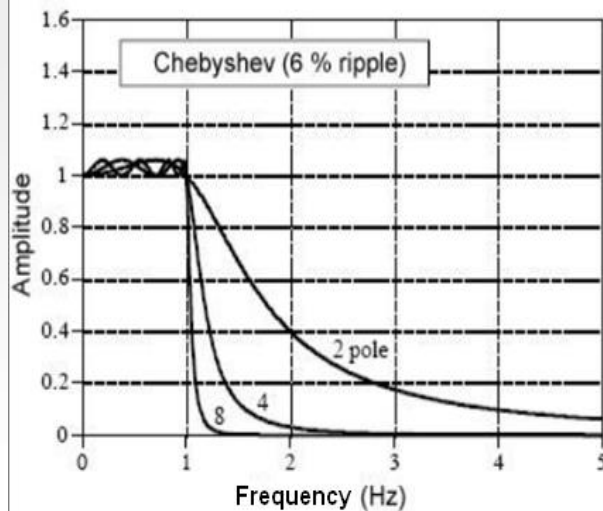
- infinite impulse response, truncated at a certain precision
- use previously calculated values from the output (recursion)
- described by recursion coefficients
- more efficient, but can be unstable





# 4.Digital Filters –Typical IIR filters

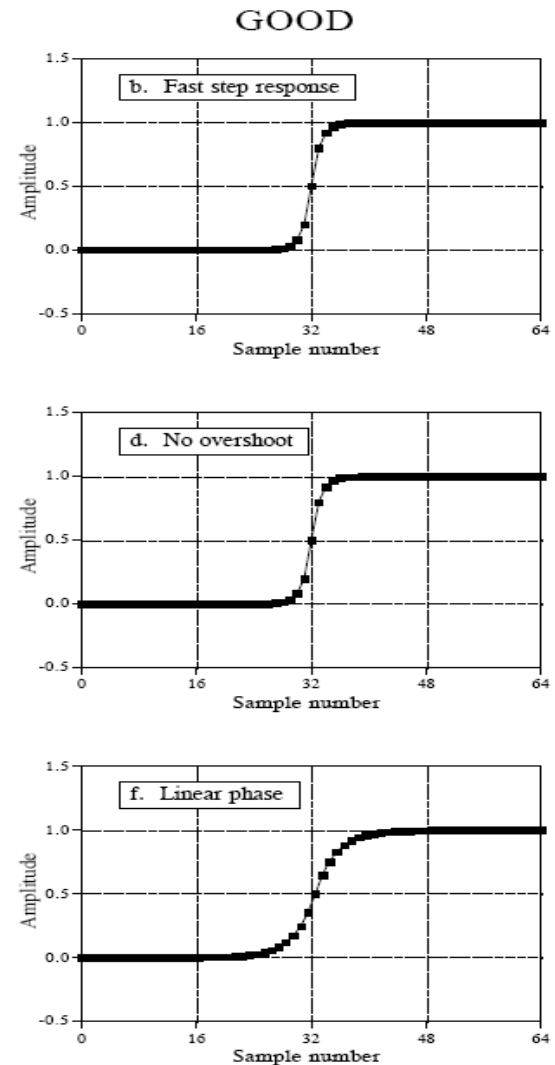
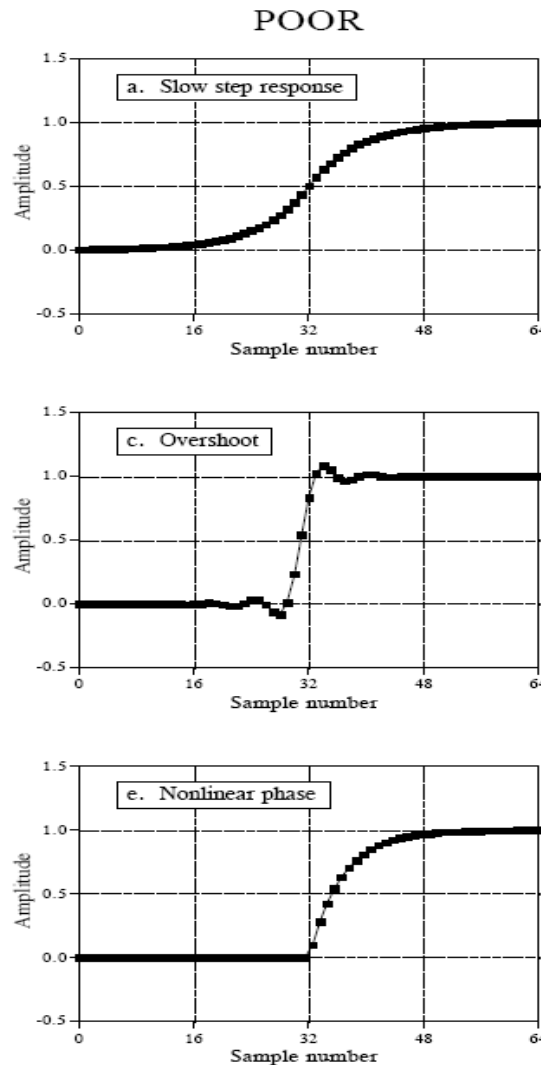
## Chebyshev, Butterworth and Bessel characteristics



- [1] Vinay K.Ingle,John G.Proakis, “ **DIGITAL SIGNAL PROCESSING Using MATLAB V.4**”, Northeastern University, 1997,  
[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, “ **Discrete-Time Signal Processing**”, Prentice-Hall, Inc.1999,1989  
[11] <http://www.dspguide.com>

# 4. Digital Filter Characteristics

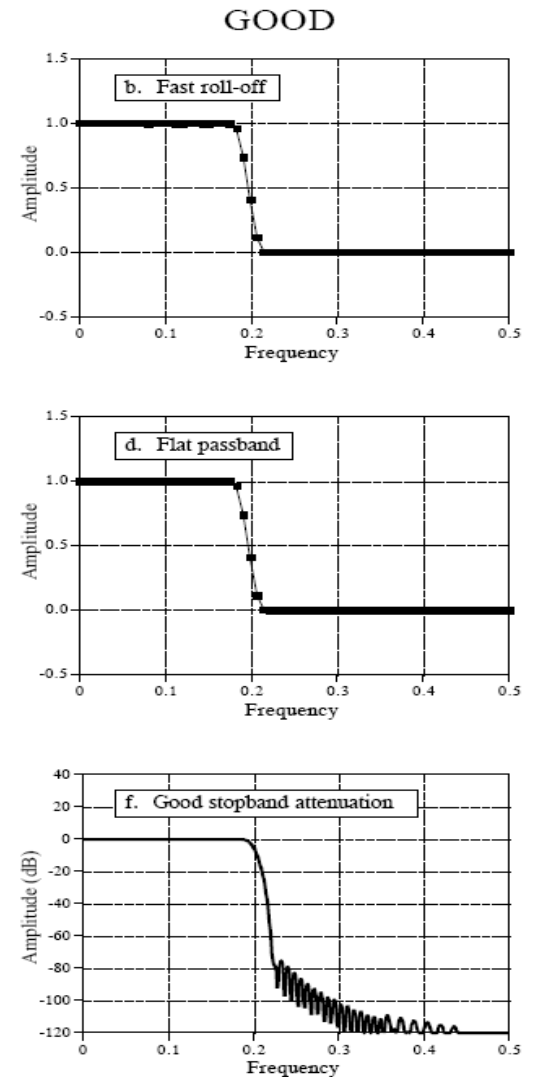
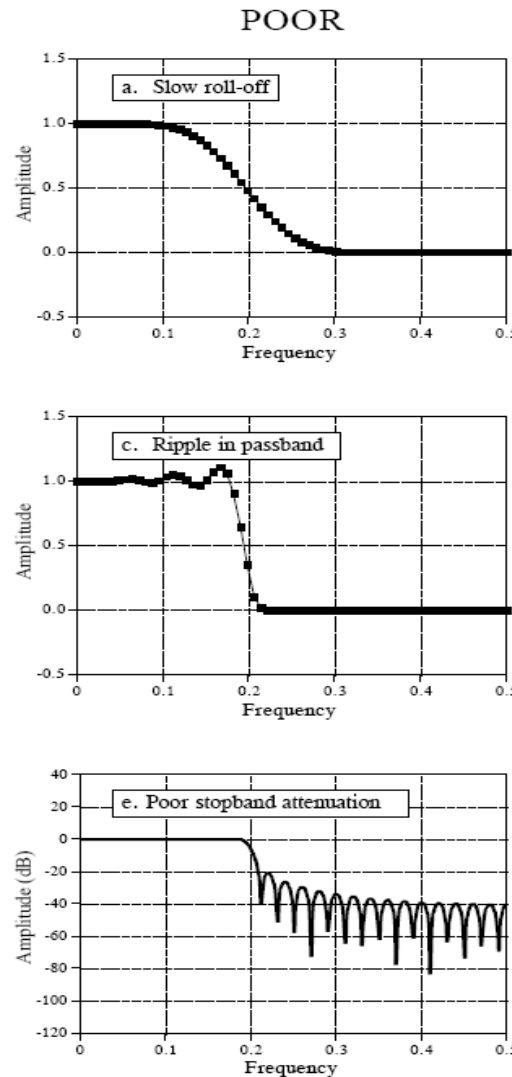
## Performance in Time Domain



- [1] Vinay K.Ingle,John G.Proakis, “ **DIGITAL SIGNAL PROCESSING Using MATLAB V.4**”, Northeastern University, 1997,  
[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, “ **Discrete-Time Signal Processing**”, Prentice-Hall, Inc.1999,1989  
[11] <http://www.dspguide.com>

# 4. Digital Filter Characteristics

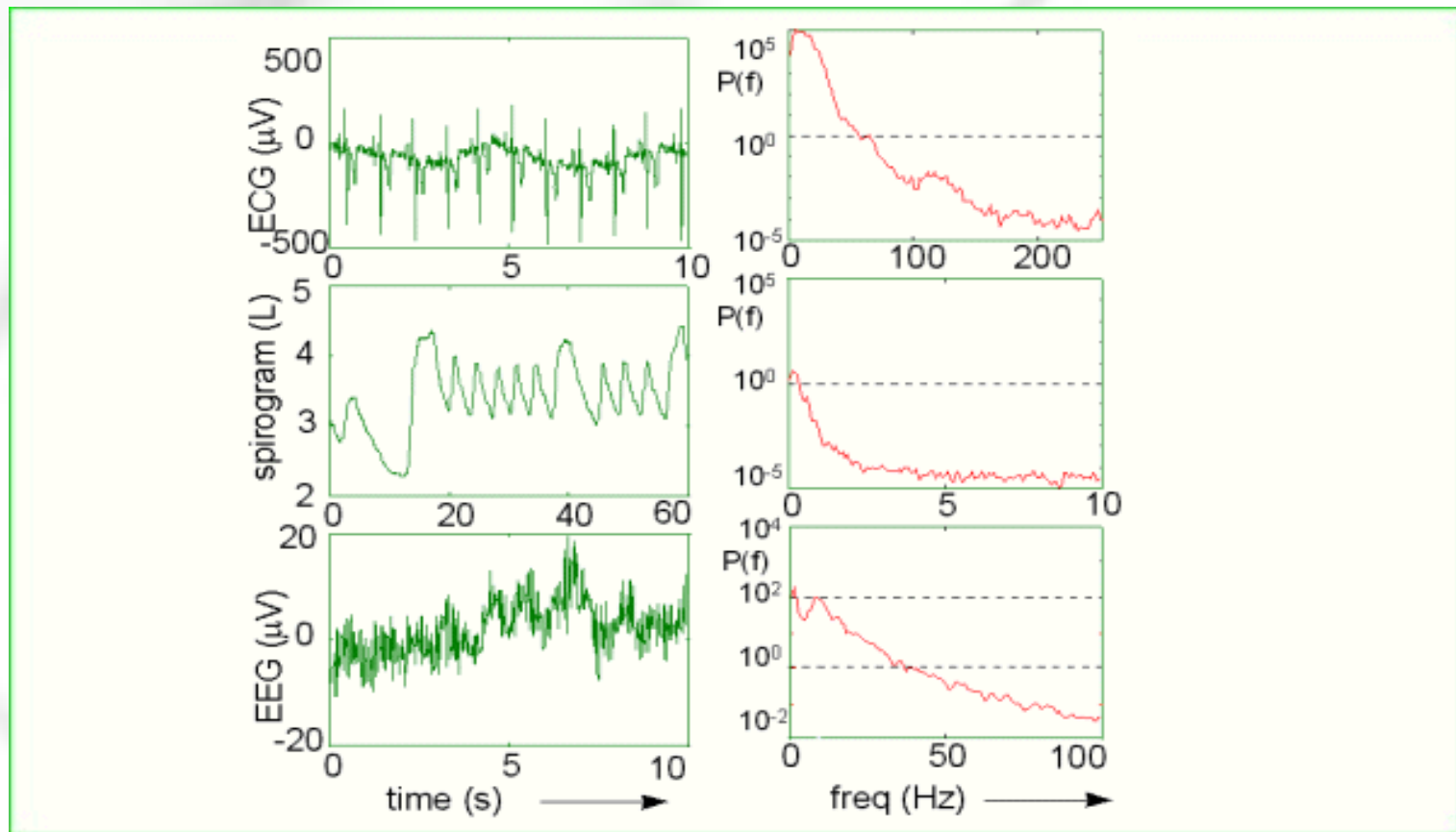
## Performance in Frequency Domain



- [1] Vinay K.Ingle,John G.Proakis, “ **DIGITAL SIGNAL PROCESSING Using MATLAB V.4**”, Northeastern University, 1997,  
[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, “ **Discrete-Time Signal Processing**”, Prentice-Hall, Inc.1999,1989  
[11] <http://www.dspguide.com>

# 4. Digital Filter Characteristics-Examples

Examples of three biological signals with their frequency spectrum-ECG,SPIROGRAM,EEG



[1] Vinay K.Ingle,John G.Proakis, “ **DIGITAL SIGNAL PROCESSING Using MATLAB V.4**”, Northeastern University, 1997,

[11] <http://www.dspguide.com>

# 4. Digital FIR Filters-

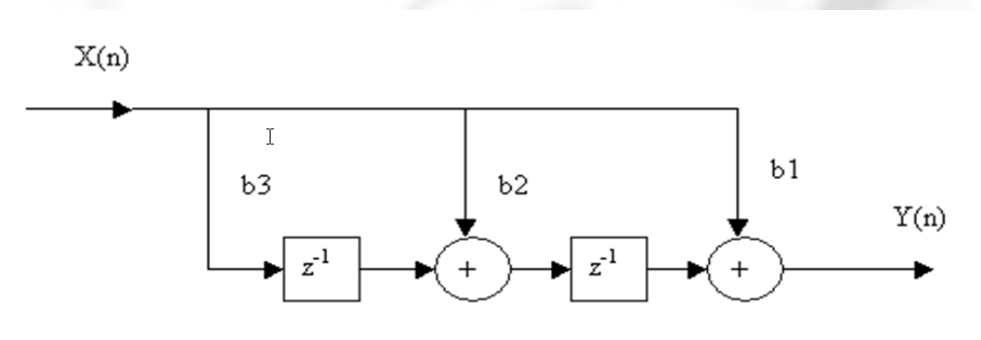
## 60 Hz notch filter example

System Function: 
$$H(z) = \frac{Y(z)}{X(z)} = \frac{(z - z_1) \cdot (z - z_2)}{z^2}$$

$$\frac{(z - z_1) \cdot (z - z_2)}{z^2} = \frac{z^2 - z \cdot z_2 - z_1 \cdot z + z_1 \cdot z_2}{z^2} = 1 - z^{-1} \cdot (z_2 + z_1) + z^{-2} \cdot z_1 \cdot z_2$$

$$1 + z^{-1}(-2(\cos(\omega_0))) + z^{-2}$$

Filter Coefficients:  $b_3=1$      $b_2=-2 \cdot \cos(\omega_0)$      $b_1=1$



$$G = \frac{1}{(2 - 2 \cos(\omega_0))}$$

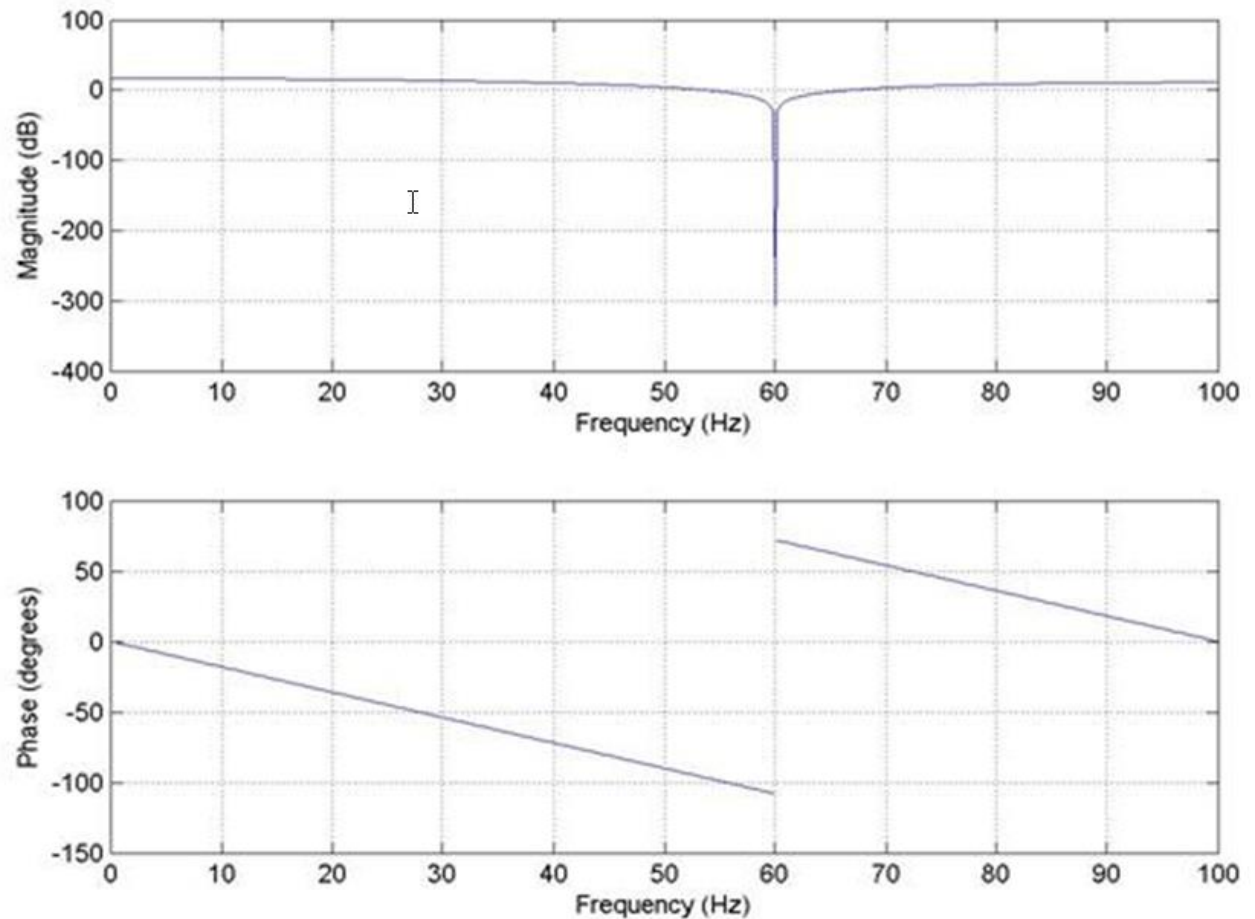
scaling:

$$G = 1.845$$

# 4. Digital FIR Filters-

## 60 Hz notch filter example

Deriving  
Filter  
Characteristics



[2] Steven T. Karris, "Signals and Systems with MATLAB Computing and Simulink Modeling", Fourth Edition, 2008

[11] <http://www.dspguide.com>

# 4. Design Digital FIR Filters - 60 Hz notch filter example

Frequencies that define complex zeros:

$$\omega_0 = 2 \cdot \pi \cdot \frac{f_0}{f_s}$$

$f_0=60\text{Hz}$  - power supply frequency

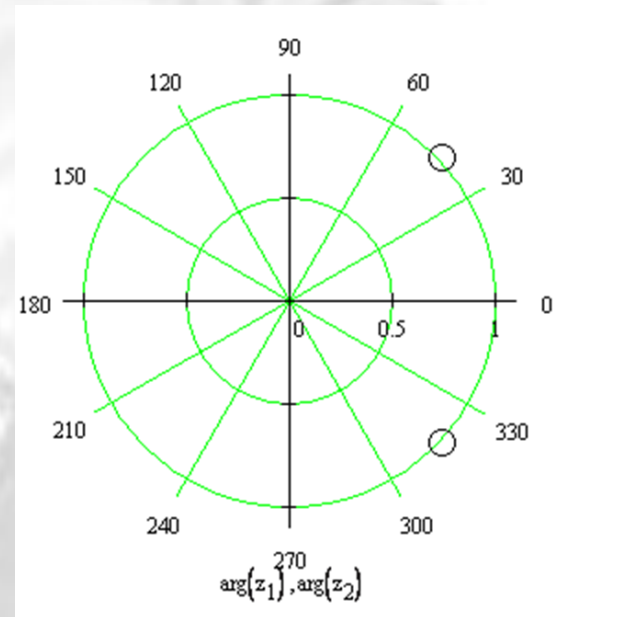
$f_s=500\text{Hz}$  - sampling rate

we get  $\omega_0 = 0.754$

**Positions of complex zeros:**

$$z_1 = \cos(\omega_0) + j \cdot \sin(\omega_0)$$

$$z_2 = \cos(\omega_0) - j \cdot \sin(\omega_0)$$

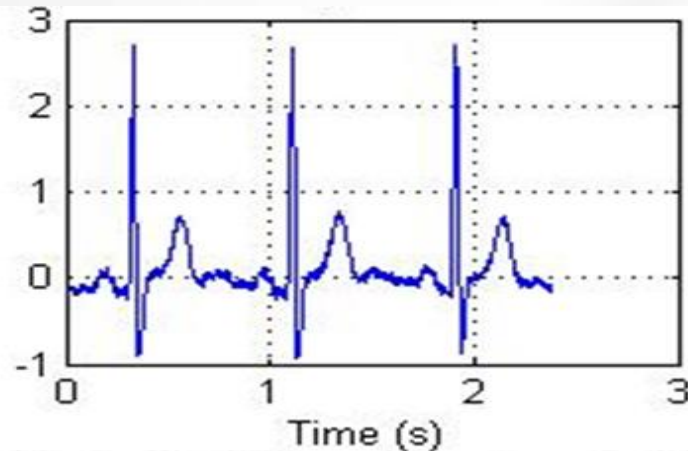


[11] <http://www.dspguide.com>

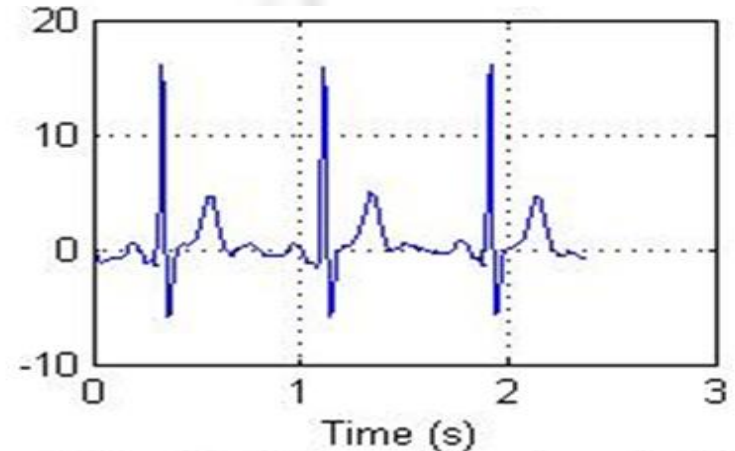
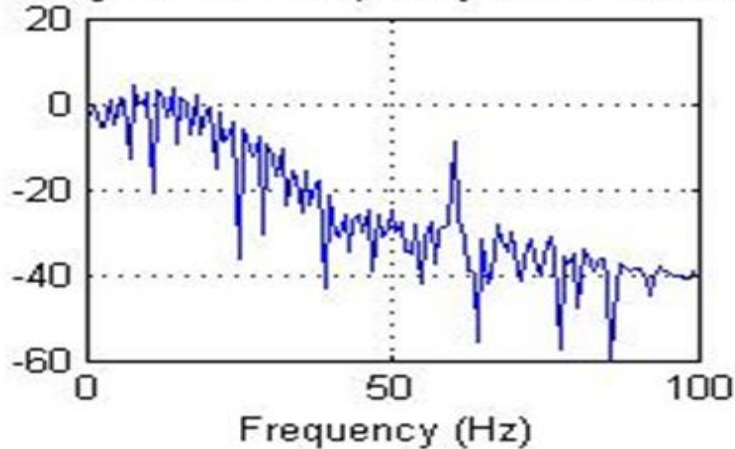
[12] Matlab-source: <http://www.scienceprog.com/category/biomedical-dsp>



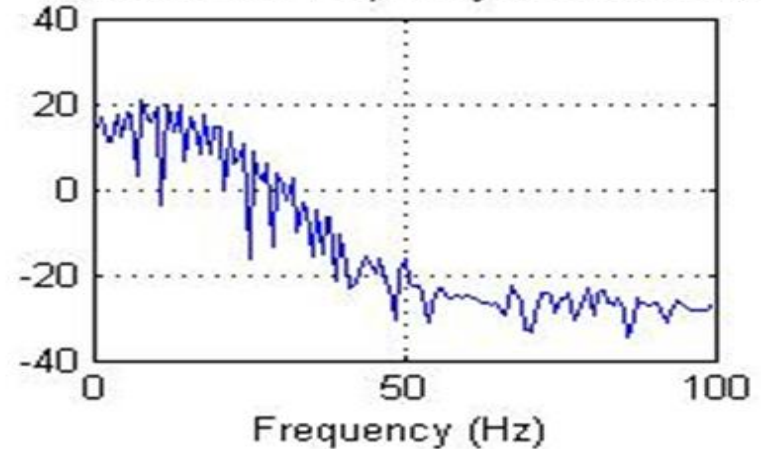
# 4. Design Digital FIR Filters - 60 Hz notch filter - Implementation in Matlab



Original ECG frequency characteristic



Filtered ECG frequency characteristic



**60Hz notch applied to ECG signal**

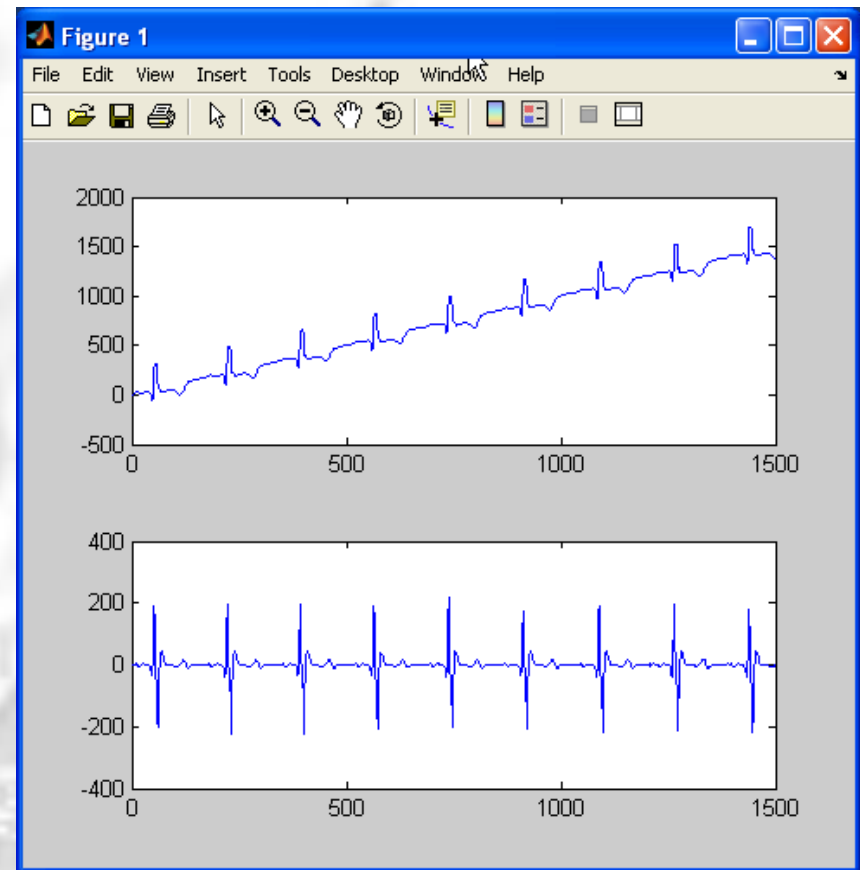
[11] <http://www.dspguide.com>

[12] Matlab-source: <http://www.scienceprog.com/category/biomedical-dsp>



# 4. Design Digital FIR Filters - 60 Hz notch filter - Implementation in Matlab

Highpass for ECG signal  
parsed from a text file



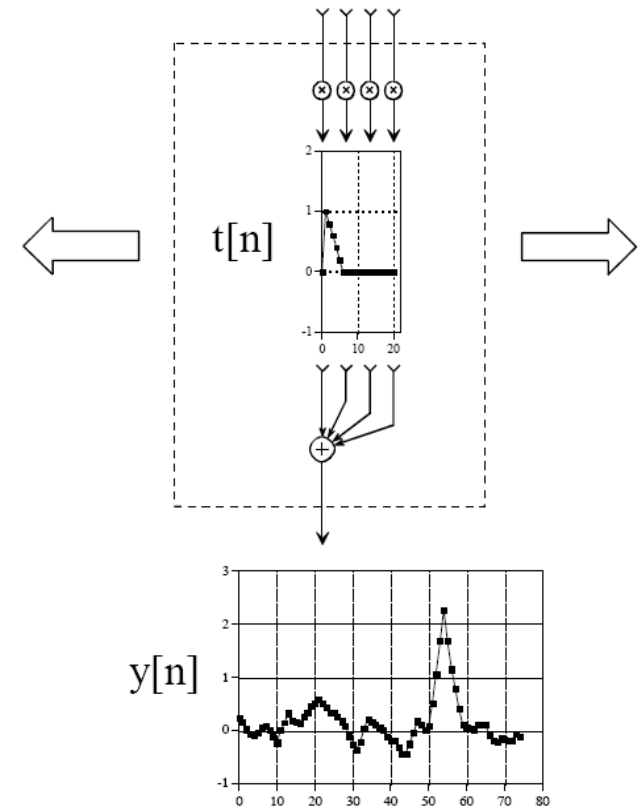
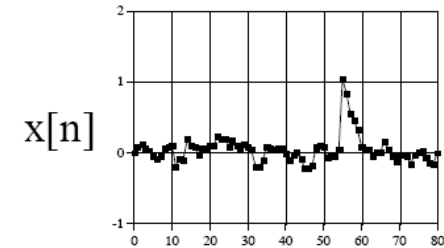
[11] <http://www.dspguide.com>

[12] Matlab-source: <http://www.scienceprog.com/category/biomedical-dsp>

# 5. Other Signal Processing Techniques

## Correlation

- mathematical operation that is very similar to convolution
- uses two signals to produce a third signal. This third signal is called the **cross-correlation** of the two input signals (i.e. finds similar signals in a signal)
- if a signal is correlated with *itself*, the resulting signal is instead called the **auto-correlation** (i.e. finds periodic parts of a signal)
- Correlation is the *optimal* technique for detecting a known waveform in random noise.



[3] Alan V. Oppenheim, Ronald W. Schaffer, John R. Buck, "Discrete-Time Signal Processing", Prentice-Hall, Inc. 1989, 1989

[11] <http://www.dspguide.com>

# 5. Other Signal Processing Techniques

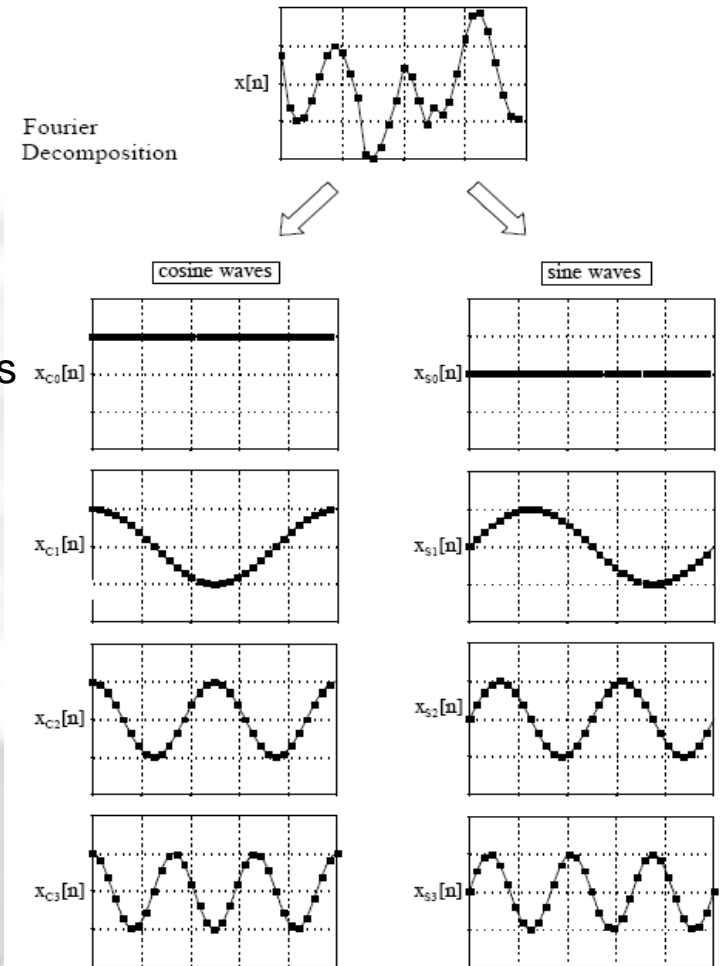
## Discrete Fourier Transform (DFT)

- Decomposition into sine- and cosine waves

$$c_k = \cos(2\pi ki / N) \quad \begin{array}{l} k \dots \text{base function} \\ i \dots \text{sample index} \end{array}$$

$$s_k = \sin(2\pi ki / N) \quad N \dots \text{number of samples}$$

- Finds frequency components of (periodic) signals
- Frequencies up to  $F/2$



[3] Alan V. Oppenheim, Ronald W. Schaffer, John R. Buck, "Discrete-Time Signal Processing", Prentice-Hall, Inc. 1999, 1989

[11] <http://www.dspguide.com>

# 5. Other Signal Processing Techniques

## Discrete Fourier Transform(DFT)

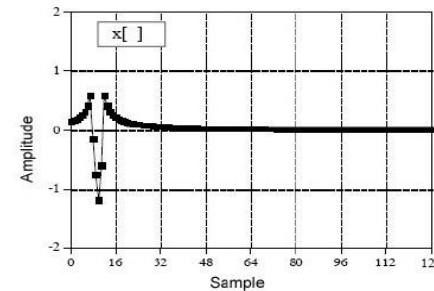
➤ Inverse Transform:

$$\text{Re } X[k] = \sum_{i=0}^{N-1} x[i] \cos(2\pi ki / N)$$

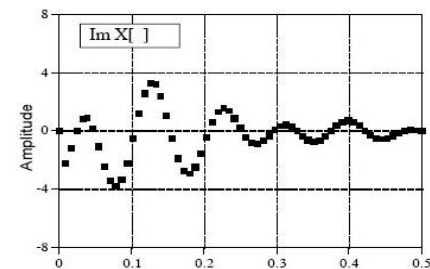
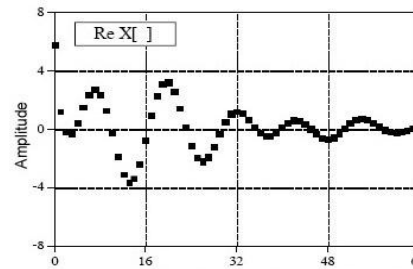
$$\text{Im } X[k] = \sum_{i=0}^{N-1} x[i] \sin(2\pi ki / N)$$

$$x[i] = \sum_{k=0}^{N/2} \text{Re } X[k] \cos(2\pi ki / N) + \sum_{k=0}^{N/2} \text{Im } X[k] \sin(2\pi ki / N)$$

➤ FFT-Algorithms



DFT



[3] Alan V. Oppenheim, Ronald W. Schaffer, John R. Buck, "Discrete-Time Signal Processing", Prentice-Hall, Inc. 1999, 1989

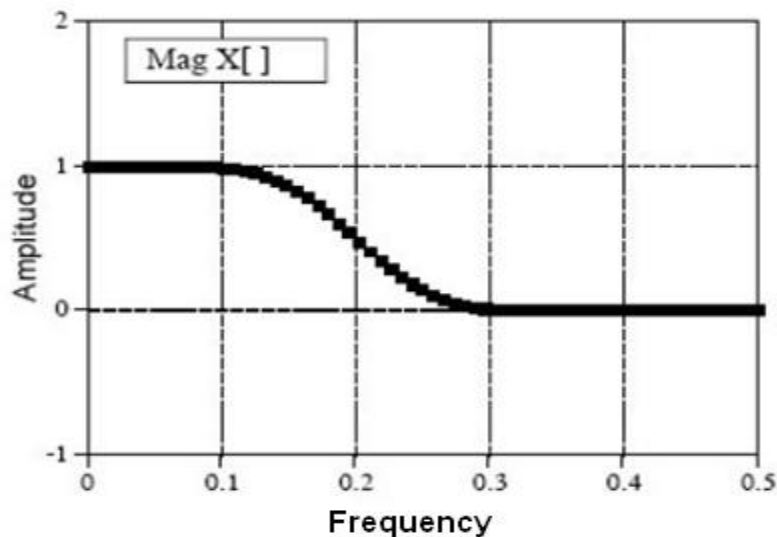
[11] <http://www.dspguide.com>

# 5. Other Signal Processing Techniques

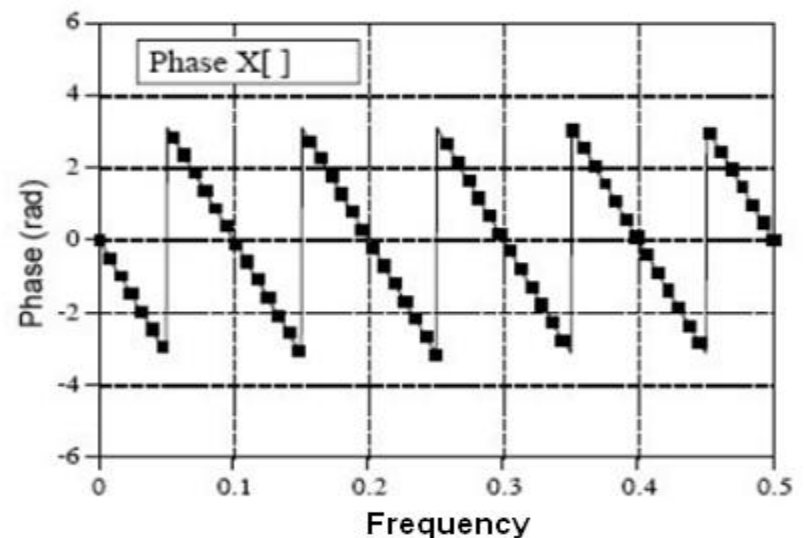
## Discrete Fourier Transform(DFT)

- Calculation of Magnitude and Phase response:

$$\text{Mag}X[k] = \sqrt{(\text{Re } X[k]^2 + \text{Im } X[k]^2)}$$



$$\text{Phase}X[k] = \arctan\left(\frac{\text{Im } X[k]}{\text{Re } X[k]}\right)$$

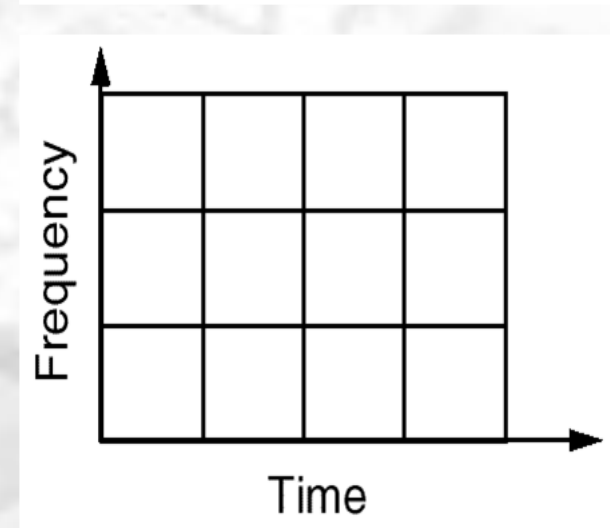
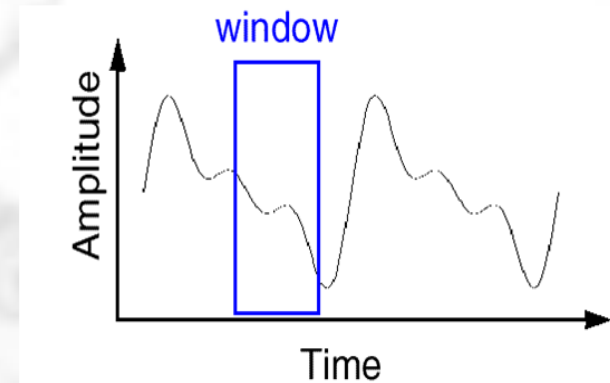


[2] Steven T.Karris, " **Signals and Systems with MATLAB Computing and Simulink Modeling**", Fourth Edition 2008

[11] <http://www.dspguide.com>

# 6. Short Time Fourier Analysis

- In order to analyze small section of a signal, Denis Gabor (1946), developed a technique, based on the FT and using windowing: STFT
- A compromise between time-based and frequency-based views of a signal.
- both time and frequency are represented in limited precision.
- The precision is determined by the size of the window.
- Once you choose a particular size for the time window - it will be the same for all frequencies.
- Many signals require a more flexible approach - so we can vary the window size to determine more accurately either time or frequency.



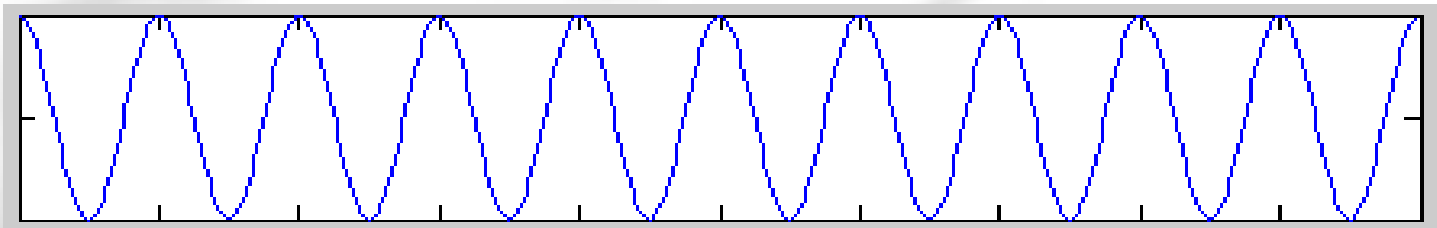
[3] Alan V. Oppenheim, Ronald W. Schaffer, John R. Buck, " **Discrete-Time Signal Processing**", Prentice-Hall, Inc. 1999, 1989

[11] <http://www.dspguide.com>

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

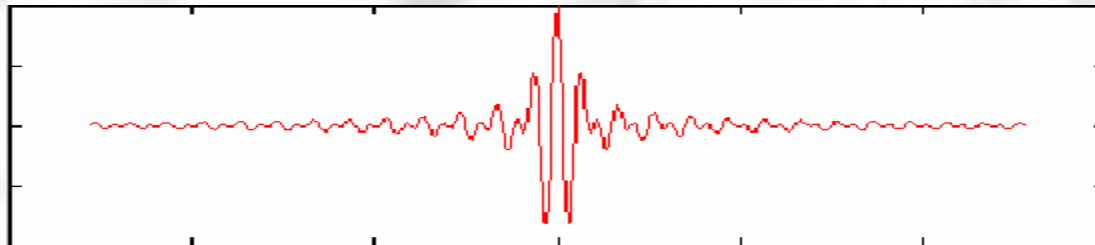
# 6. Fourier Analysis-Wavelet Analysis

- Fourier Analysis is based on an indefinitely long cosine wave of a specific Frequency



time,  $t$

- Wavelet Analysis is based on a short duration wavelet of a specific center frequency



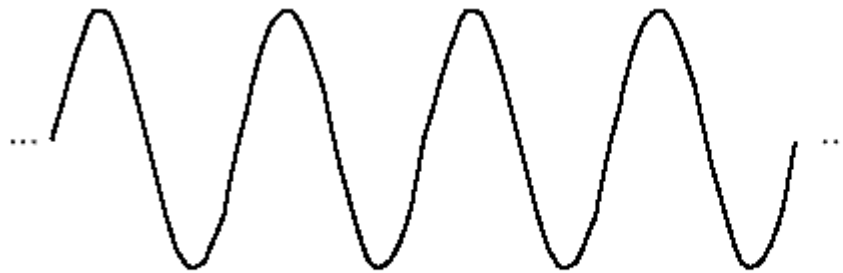
time,  $t$

[2] Steven T.Karris, “ **Signals and Systems with MATLAB Computing and Simulink Modeling**”, Fourth Edition, 2008

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.

# 6. What is Wavelet Analysis ?

- A wavelet is a waveform of effectively limited duration that has an average value of zero.



Sine Wave



Wavelet (db10)



# 6. Wavelet's properties

- Short time localized waves with zero integral value.
- Possibility of time shifting.
- Flexibility.

# 6.The Continuous Wavelet Transform (CWT)

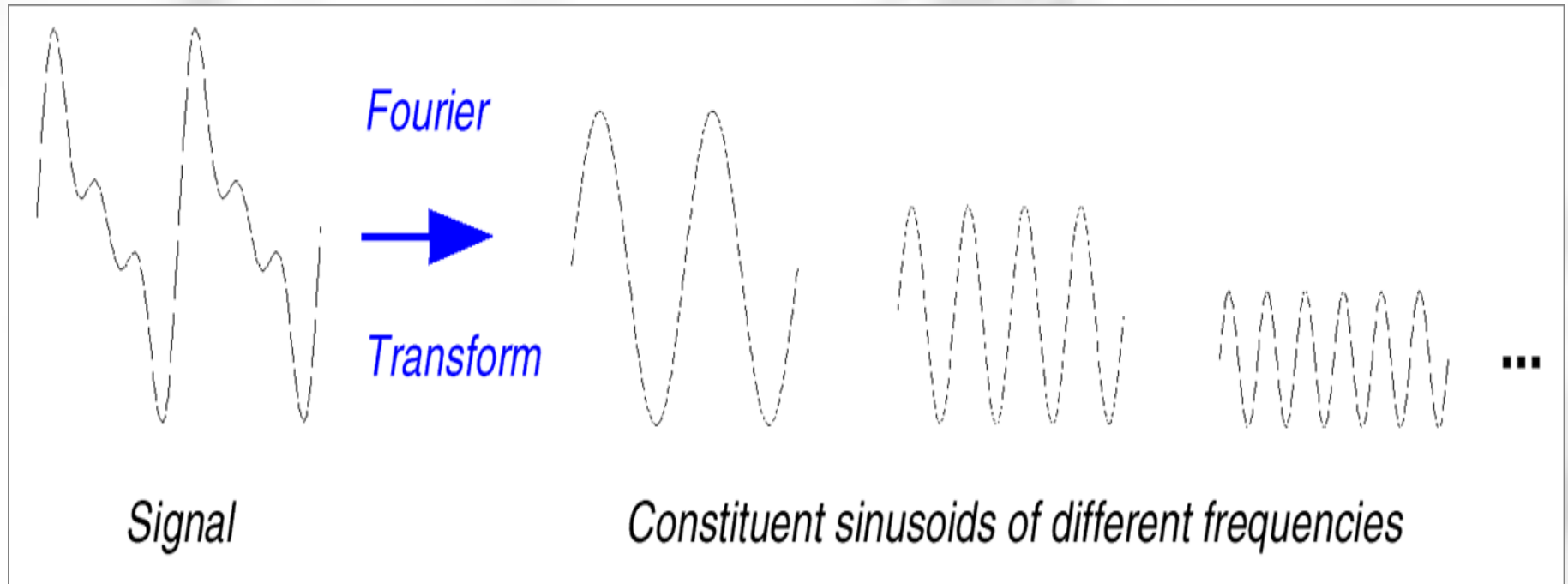
A mathematical representation of the Fourier transform:

$$F_w = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} \cdot dt$$

the sum over all time of the signal  $f(t)$  multiplied by a complex exponential, and the result is the Fourier coefficients  $F(\omega)$  .

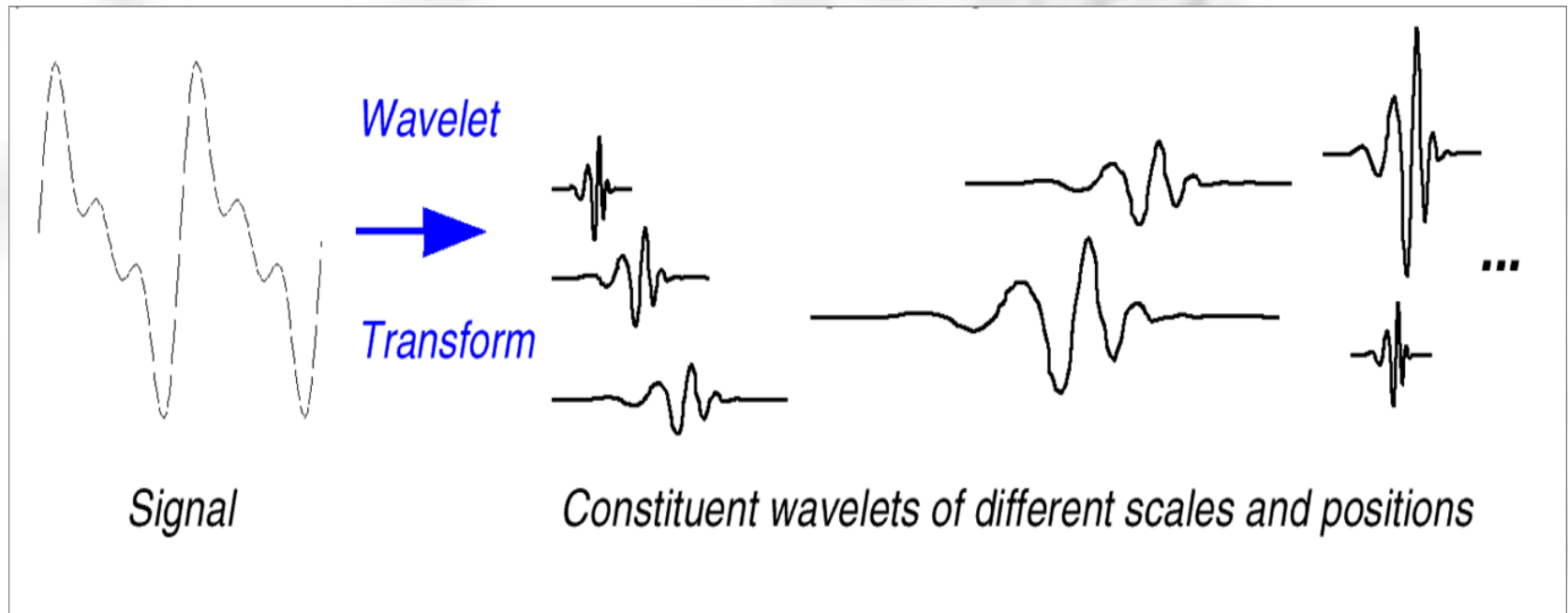
# 6. Wavelet Transformation

Those coefficients, when multiplied by a sinusoid of appropriate frequency, yield the constituent sinusoidal component of the original signal:



# 6. Wavelet Transformation

- And the result of the CWT are Wavelet coefficients
- Multiplying each coefficient by the appropriately scaled and shifted wavelet yields the constituent wavelet of the original signal:

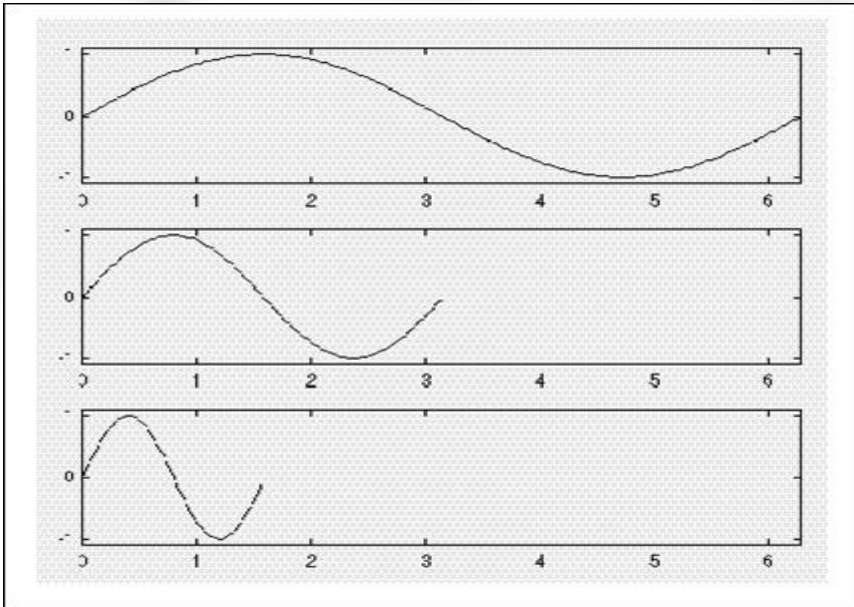


# 6. Wavelet Transformation- Equations

- Wavelet Transform  $\gamma(s, \tau) = \int f(t) \psi_{s,T}^*(t) dt$
- Inverse Wavelet Transform  $f(t) = \iint \gamma(s, \tau) \psi_{s,T}(t) d\tau ds$
- All wavelet derived from *mother wavelet*  $\psi_{s,T}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right)$

# 6. Wavelet Transformation-Scaling

- Wavelet analysis produces a time-scale view of the signal.
- *Scaling* means stretching or compressing of the signal.
- scale factor ( $a$ ) for sine waves:



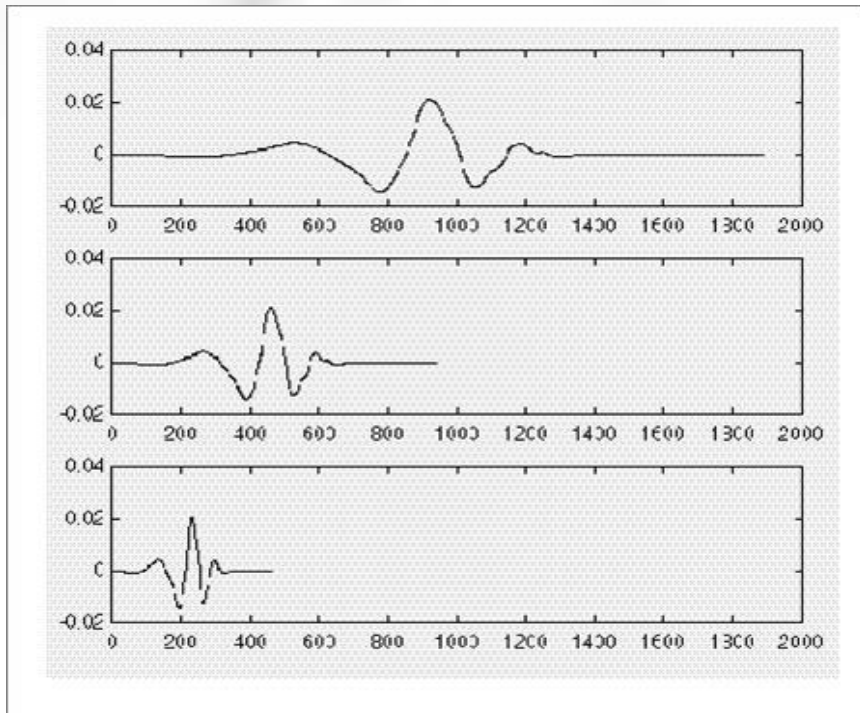
$$f(t) = \sin(t); a = 1$$

$$f(t) = \sin(2t); a = \frac{1}{2}$$

$$f(t) = \sin(4t); a = \frac{1}{4}$$

# 6. Wavelet Transformation-Scaling

- Scale factor works exactly the same with wavelets:



$$f(t) = \Psi(t); a = 1$$

$$f(t) = \Psi(2t); a = \frac{1}{2}$$

$$f(t) = \Psi(4t); a = \frac{1}{4}$$



# 6. Wavelet Transformation- Wavelet function

$$\Psi_{a,b}(x) = \frac{1}{\sqrt{a}} \Psi\left(\frac{x-b}{a}\right)$$

- $b$ - shift coefficient
- $a$ - scale coefficient

$$\Psi_{a,b_x,b_y}(x,y) = \frac{1}{|a|} \Psi\left(\frac{x-b_x}{a}, \frac{y-b_y}{a}\right)$$

- 2D function

# 6. Wavelet Transformation- Wavelet function

$$\psi_{s,T}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right)$$

normalization

shift in time

change in scale:  
big s means long  
wavelength

Mother wavelet

wavelet with  
scale, s and time,  $\tau$

# 6. Wavelet Transformation

$$\gamma(s, \tau) = \int f(t) \psi_{s, \tau}^*(t) dt$$

time-series

coefficient of wavelet with scale,  $s$  and time,  $\tau$

complex conjugate of wavelet with scale,  $s$  and time,  $\tau$

I'm going to ignore the complex conjugate from now on, assuming that we're using real wavelets

# 6. Wavelet Transformation-Wavelets

## examples

### Dyadic transform

- For easier calculation we can to discrete continuous signal.
- We have a grid of discrete values that called dyadic grid .
- Important that wavelet functions compact (e.g. no over-calculatings)

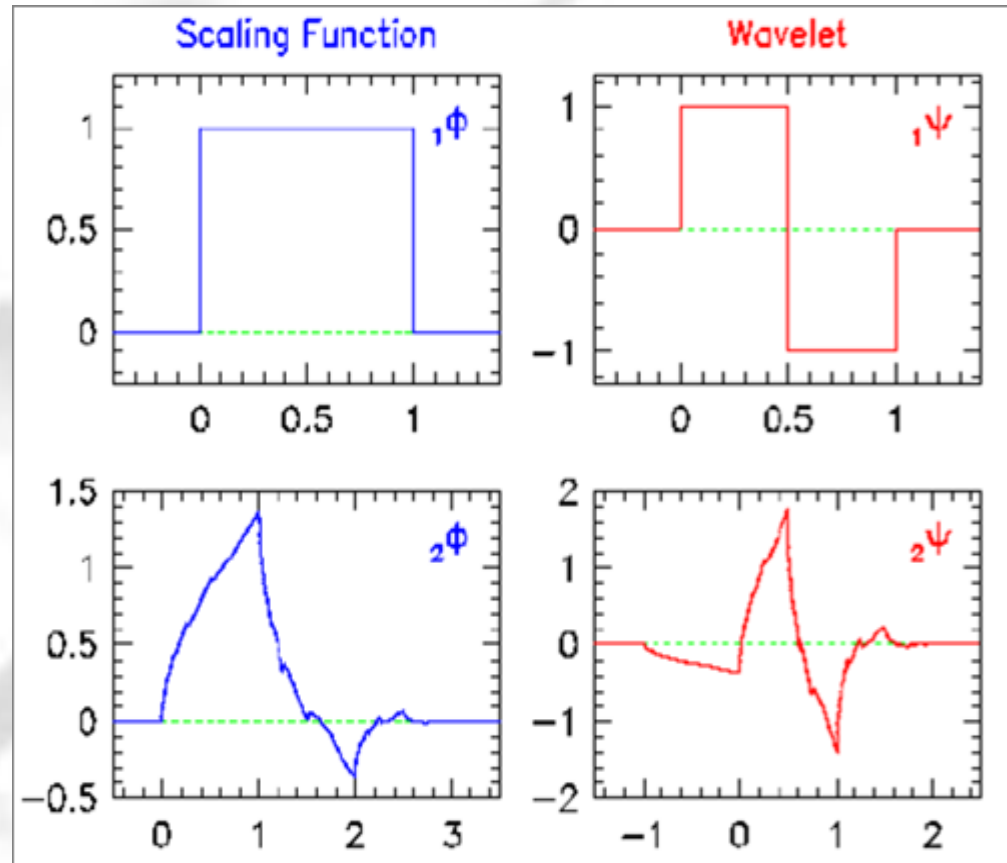
$$a = 2^j$$

$$b = k 2^j$$

# 6. Wavelet Transformation- Wavelet functions examples

➤ Haar function

➤ Daubechies function



# 6. Inverse Wavelet Transform

$$f(t) = \iint \gamma(s, \tau) \psi_{s, \tau}(t) d\tau ds$$

time-series

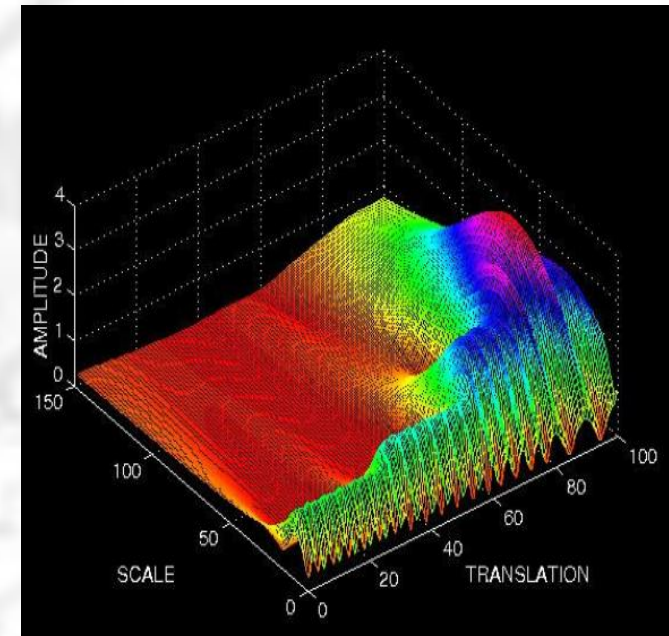
coefficients  
of wavelets

wavelet with  
scale,  $s$  and time,  $\tau$

build up a time-series as sum of wavelets of different scales,  $s$ , and positions,  $t$

# 6. Wavelet Transformation

- good frequency resolution at low frequencies and
  - good time resolution at high frequencies
  - no work-around for the principle of entropy
- 
- scale (s) and translation (t) of the base wavelet
  - convolution with the signal
  - special wavelets for special purposes



[11] <http://www.dspguide.com>

[17] Martin Vetterli and Jelena Kovacevic, *Wavelets and Subband Coding*. Prentice Hall, 1995.



# 7.Data mining

- The use of tools to extract useful information & patterns in bodies of data for use in decision support and estimation
- The automated extraction of hidden predictive information from (large) databases

## Diagnosis:

Recognize and classify patterns in multivariate patient attributes

## Therapy:

Select from available treatment methods; based on effectiveness, suitability to patient, etc.

## Prognosis:

Predict future outcomes based on previous experience and present conditions

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

[16] I. H. Witten and E. Frank. **Data Mining: Practical Machine Learning Tools and Techniques**, 2ed. Morgan Kaufmann, 2005.

# 7.DATA MINING- Methods

## ➤ FUNCTIONS-METHODS:

- Clustering into 'natural' groups (unsupervised)
- Classification into known classes; e.g. diagnosis (supervised)
- Detection of associations
- Detection of sequential temporal patterns; e.g. disease development
- Prediction or estimation of an outcome
- Time series forecasting

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

[16] I. H. Witten and E. Frank. **Data Mining: Practical Machine Learning Tools and Techniques**, 2ed. Morgan Kaufmann, 2005.

# 7.DATA MINING-Supervised vs. Unsupervised Learning

## ➤ Supervised learning (classification)

- Supervision: The training data (observations, measurements, etc.) are accompanied by labels indicating the class of the observations
- New data is classified based on the training set

## ➤ Unsupervised learning (clustering)

- The class labels of training data is unknown
- Given a set of measurements, observations, etc. the aim is to establish the existence of classes or clusters in the data

# 7.DATA MINING-Classification

Simple classification could be based on:

- Thresholds (levels / intervals, adaptive thresholds)
- absolute values + averaging over intervals
- integration / difference
- local minima / maxima, zeros in time domain
- Energy, energy distribution over frequency bands

# 7.DATA MINING- Bayesian Classification

## Bayesian Theorem

- Given training data  $D$ , *posteriori* probability of a hypothesis  $h$ ,  $P(h|D)$  follows the Bayes theorem

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- MAP (maximum posteriori) hypothesis

$$h_{MAP} \equiv \arg \max_{h \in H} P(h|D) = \arg \max_{h \in H} P(D|h)P(h).$$

- Practical difficulties:

- require initial knowledge of many probabilities
- significant computational cost

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

[16] I. H. Witten and E. Frank. **Data Mining: Practical Machine Learning Tools and Techniques**, 2ed. Morgan Kaufmann, 2005.

# 7.DATA MINING- Bayesian Classification

- The classification problem may be formalized using a-posteriori probabilities:
- $P(C|X)$  = prob. that the sample tuple  $X=\langle x_1, \dots, x_k \rangle$  is of class C.
- e.g.  $P(\text{class}=N \mid \text{outlook}=\text{sunny}, \text{windy}=\text{true}, \dots)$

# 7.DATA MINING-NEURAL NETWORKS

**A set of connected input/output units where each connection has a weight associated with it**

## ➤ Advantages

- prediction accuracy is generally high
- robust, works when training examples contain errors
- output may be discrete, real-valued, or a vector of several discrete or real-valued attributes
- fast evaluation of the learned target function

## ➤ Disadvantages

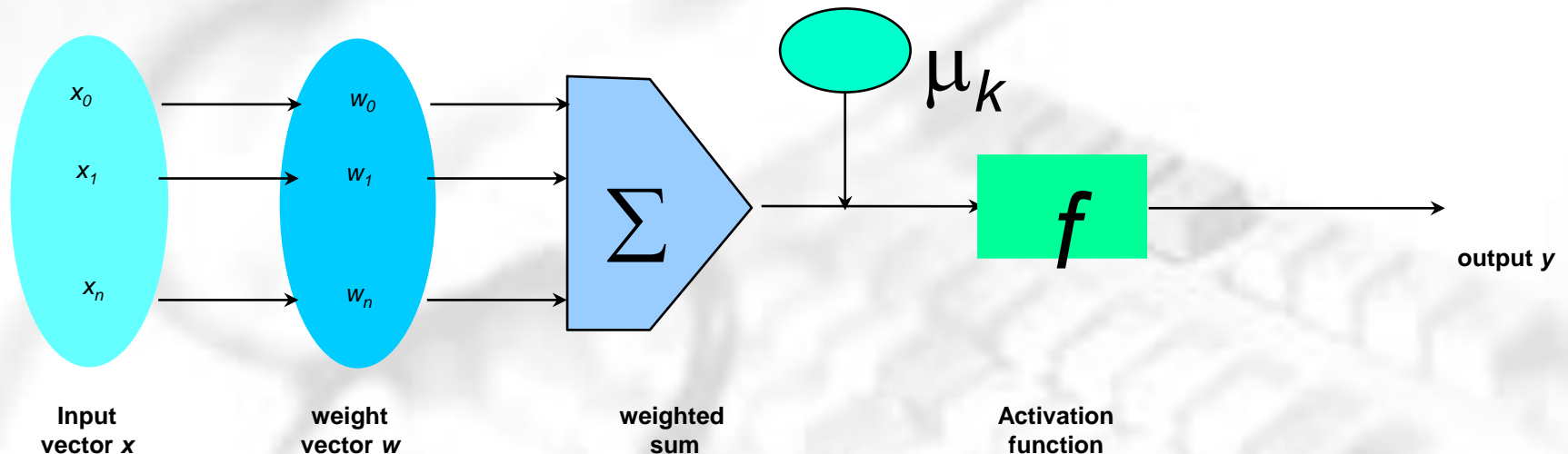
- long training time
- require (typically empirically determined) parameters (e.g. network topology)
- difficult to understand the learned function (weights)
- not easy to incorporate domain knowledge

[11] <http://www.dspguide.com/>

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

# 7.DATA MINING-NEURAL NETWORKS

## A Neuron



- The  $n$ -dimensional input vector  $x$  is mapped into variable  $y$  by means of the scalar product and a nonlinear function mapping

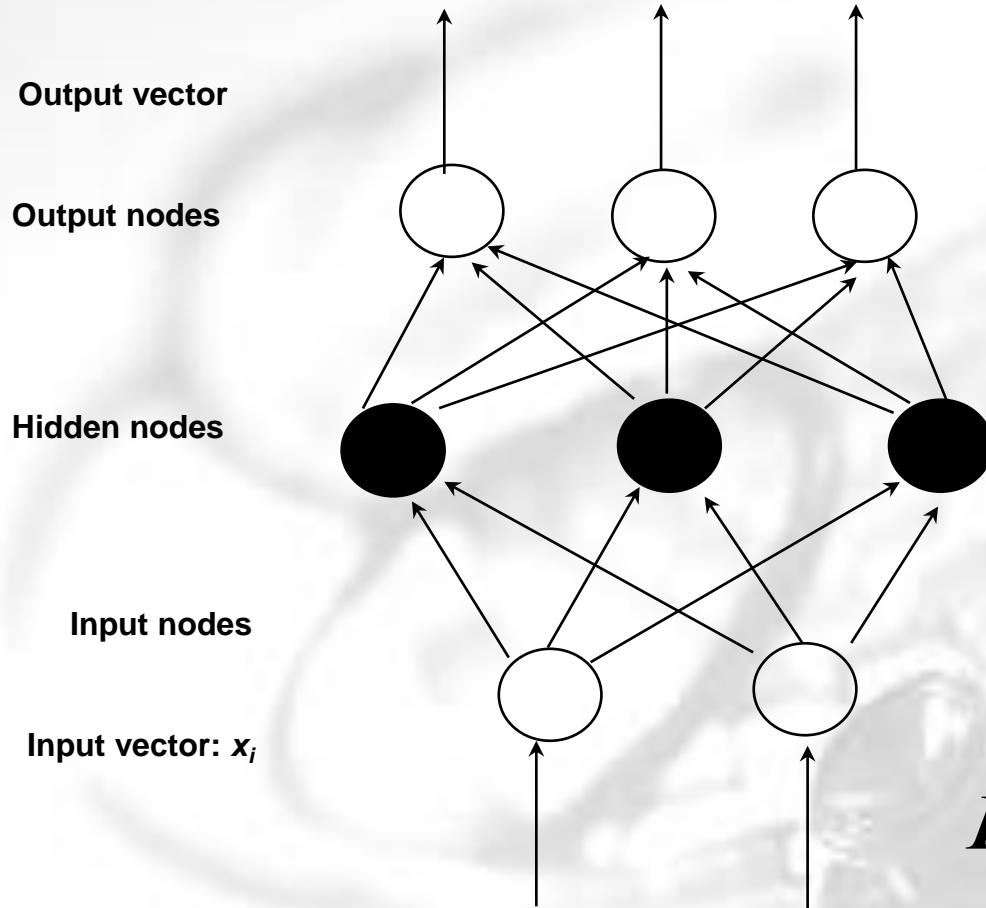
[11] <http://www.dspguide.com/>

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.



# 7.DATA MINING-NEURAL NETWORKS

## Multi-Layer Perceptron



$$Err_j = O_j(1 - O_j) \sum_k Err_k w_{jk}$$

$$\theta_j = \theta_j + (l) Err_j$$

$$w_{ij} = w_{ij} + (l) Err_j O_i$$

$$Err_j = O_j(1 - O_j)(T_j - O_j)$$

$$O_j = \frac{1}{1 + e^{-I_j}}$$

$$I_j = \sum_i w_{ij} O_i + \theta_j$$

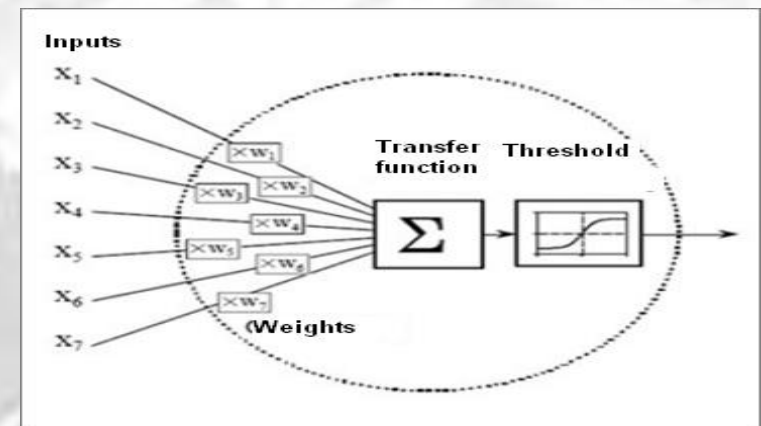
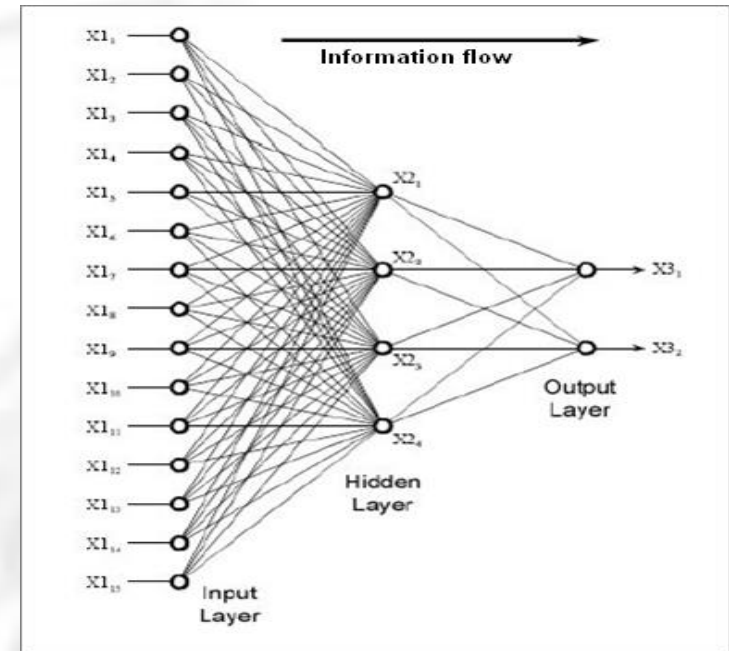
[11] <http://www.dspguide.com/>

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

# 7.DATA MINING-NEURAL NETWORKS

## Neural Networks:

- mimic biological signal processing
- Input-, hidden and output-layers units with activation functions
- learning algorithms e.g.  
error back propagation  
unsupervised learning / clustering
- internal representation unrevealed
- pattern recognition, prediction



[11] <http://www.dspguide.com/>

[16] I. H. Witten and E. Frank. **Data Mining: Practical Machine Learning Tools and Techniques**, 2ed. Morgan Kaufmann, 2005.

# 7.DATA MINING-SVMs-Support Vector Machines

- A new classification method for both linear and nonlinear data
- It uses a nonlinear mapping to transform the original training data into a higher dimension
- With the new dimension, it searches for the linear optimal separating hyperplane (i.e., “decision boundary”)
- With an appropriate nonlinear mapping to a sufficiently high dimension, data from two classes can always be separated by a hyperplane
- SVM finds this hyperplane using support vectors (“essential” training tuples) and margins (defined by the support vectors)

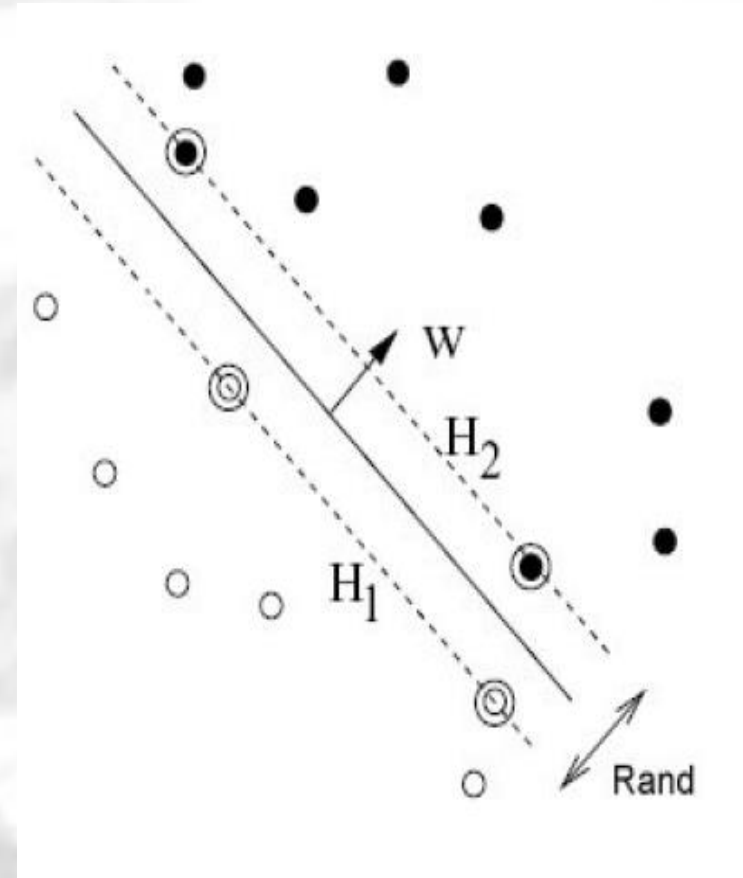
[11] <http://www.dspguide.com/>

[16] I. H. Witten and E. Frank. **Data Mining: Practical Machine Learning Tools and Techniques**, 2ed. Morgan Kaufmann, 2005.

# 7.DATA MINING - SVMs

## Support Vector Machines (SVMs):

- binary classification of an input vector
- training with classified data separates the feature space in two areas, with maximal distance of positive / negative classifications
- SVMs find a global minimum (in contrast to e.g. neural networks)



[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

[16] I. H. Witten and E. Frank. **Data Mining: Practical Machine Learning Tools and Techniques**, 2ed. Morgan Kaufmann, 2005.

# 7.DATA MINING-SVMs-Kernel functions

- Instead of computing the dot product on the transformed data tuples, it is mathematically equivalent to instead applying a kernel function  $K(\mathbf{X}_i, \mathbf{X}_j)$  to the original data, i.e.,  $K(\mathbf{X}_i, \mathbf{X}_j) = \Phi(\mathbf{X}_i) \cdot \Phi(\mathbf{X}_j)$

- Typical Kernel Functions

Polynomial kernel of degree  $h$ :  $K(X_i, X_j) = (X_i \cdot X_j + 1)^h$

Gaussian radial basis function kernel:  $K(X_i, X_j) = e^{-\|X_i - X_j\|^2 / 2\sigma^2}$

Sigmoid kernel:  $K(X_i, X_j) = \tanh(\kappa X_i \cdot X_j - \delta)$

- SVM can also be used for classifying multiple ( $> 2$ ) classes and for regression analysis (with additional user parameters)

[11] <http://www.dspguide.com/>

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

[16] I. H. Witten and E. Frank. **Data Mining: Practical Machine Learning Tools and Techniques**, 2ed. Morgan Kaufmann, 2005.

# 7.DATA MINING-SVMs vs. Neural Network

## • SVM

- Relatively new concept
- Deterministic algorithm
- Nice Generalization properties
- Hard to learn – learned in batch mode using quadratic programming techniques
- Using kernels can learn very complex functions

## • Neural Network

- Relatively old
- Nondeterministic algorithm
- Generalizes well but doesn't have strong mathematical foundation
- Can easily be learned in incremental fashion
- To learn complex functions—use multilayer perceptron (not that trivial)

[11] <http://www.dspguide.com/>

[15] C. M. Bishop, **Neural Networks for Pattern Recognition**. Oxford University Press, 1995.

# 8. Basic Signal Statistics

- Sensitivity
- Specificity
- Positive Predictive Value
- Negative Predictive Value
- Likelihood Ratio
- Relative Risk
- Absolute Risk
- Number needed to treat/harm



# 8. Basic Signal Statistics- Sensitivity and Specificity

- Four possible situations:

		Condition is:		
		Present	Absent	
Test Result:	Present	True Positive	False Positive	This is Total # of “positive” tests
	Absent	False Negative	True Negative	This is Total # of “negative” tests



# 8. Basic Signal Statistics- Sensitivity and Specificity

- *Sensitivity* is the proportion of condition present cases on which the test returned “positive”
- Analogous to the hit rate (H) in Signal Detection Theory

$$Sensitivity = \frac{\# \text{ True Positives}}{\# \text{ True Positives} + \# \text{ False Negatives}}$$

- *Specificity* is the proportion of condition absent cases on which the test returned “negative”
- Analogous to the Correct Rejection rate in Signal Detection Theory

$$Specificity = \frac{\# \text{ True Negative}}{\# \text{ True Negative} + \# \text{ False Positive}}$$

Sensitivity and Specificity have a similar relationship: as a cut-off value for a test becomes more stringent the sensitivity goes down and the specificity goes up...and vice versa

[4 ] Christopher M.Bishop, “ **Pattern Recognition and Machine Learning**”, Springer, 2006

[18] Kirkwood BR. Essentials of medical statistics. Oxford, Blackwell Science, 1988.

# 8. Basic Signal Statistics

- **Likelihood Ratio** is the ratio of True Positive *rate* to False Positive *rate*

$$\text{Likelihood Ratio} = \frac{\text{Sensitivity}}{1 - \text{Specificity}}$$

- If a test is positive, how likely is it that the condition is present?

**Positive Predictive Value** is the proportion of “positive” test results that are correct

$$PPV = \frac{\# \text{ True Positives}}{\# \text{ True Positives} + \# \text{ False Positives}}$$

- **Negative Predictive Value** is the proportion of “negative” test results that are correct

$$NPV = \frac{\# \text{ True Negatives}}{\# \text{ True Negatives} + \# \text{ False Negatives}}$$

[4 ] Christopher M.Bishop, “ **Pattern Recognition and Machine Learning**”, Springer, 2006

[18] Kirkwood BR. Essentials of medical statistics. Oxford, Blackwell Science, 1988.

# 8. Basic Signal Statistics

- **Relative Risk** is the ratio of Exposure Events to Non-Exposure Events

$$\text{Exposure Event Rate} = \frac{A}{A + B}$$

$$\text{Control Event Rate} = \frac{C}{C + D}$$

$$\text{Relative Risk} = \frac{\text{Exposure Event Rate}}{\text{Control Event Rate}} = \frac{A/(A + B)}{C/(C + D)}$$

- **Relative Risk Reduction** is the difference between event rates in the exposure and non-exposure groups, expressed as a fraction of the non-exposure event rate (it can be positive or negative)

$$\text{Relative Risk Reduction} = \frac{\text{Exposure Event Rate} - \text{Control Event Rate}}{\text{Control Event Rate}}$$

[4] Christopher M. Bishop, “**Pattern Recognition and Machine Learning**”, Springer, 2006

[18] Kirkwood BR. **Essentials of medical statistics**. Oxford, Blackwell Science, 1988.

# 8. Basic Signal Statistics

- The **absolute risk reduction** conveys effect size

$$\text{Absolute Risk Reduction} = |\text{Exposure Rate} - \text{Control Rate}|$$

- An intuitive version is to consider the reciprocal - the “**number needed to treat or harm**”

$$\text{Number Needed to Treat or Harm} = \frac{1}{\text{Absolute Risk Reduction}}$$

- Indicates the number of individuals that would have to be exposed to the treatment in order to cause one to have the outcome of interest

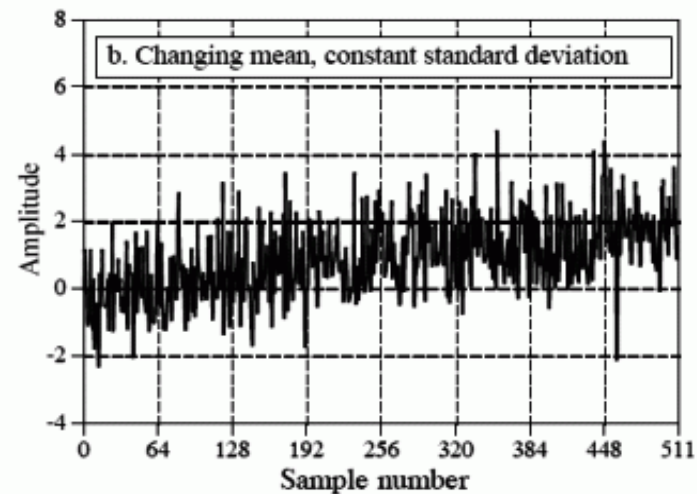
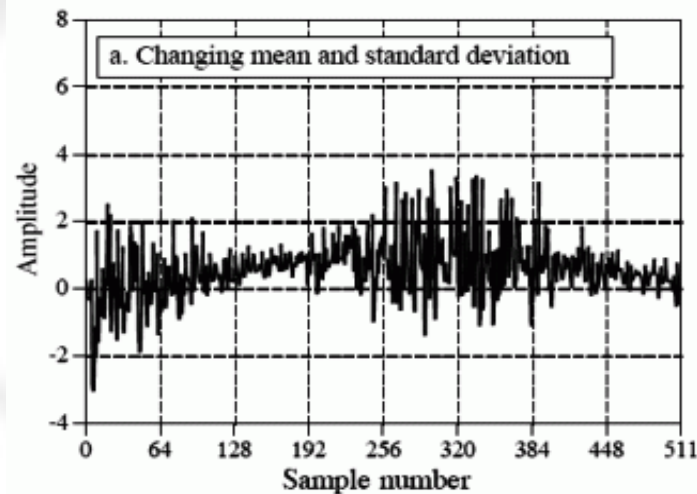
# 8. Basic Signal Statistics

Mean:

$$\mu = \frac{1}{N} \sum_{i=0}^{N-1} X_i$$

Standard deviation:

$$\sigma^2 = \frac{1}{N-1} \sum_{i=0}^{N-1} (X_i - \mu)^2$$

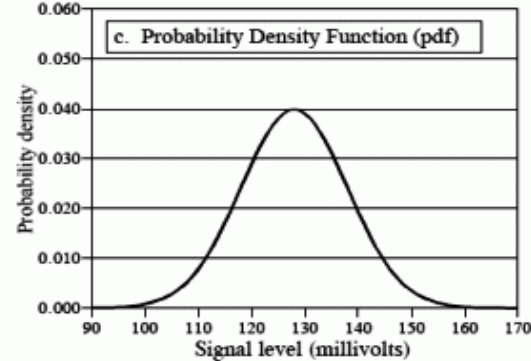
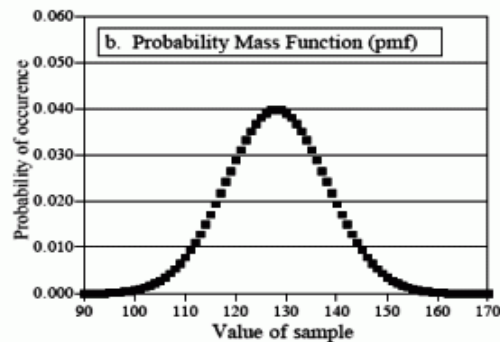
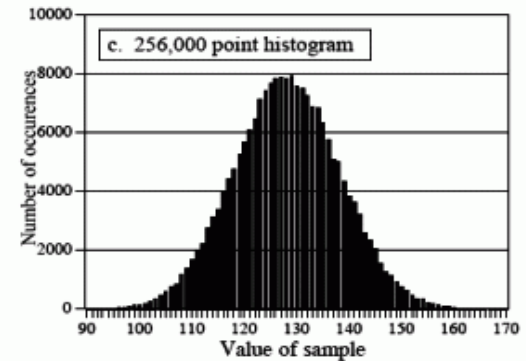
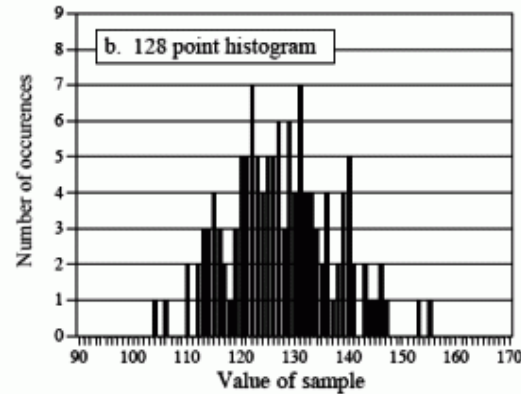
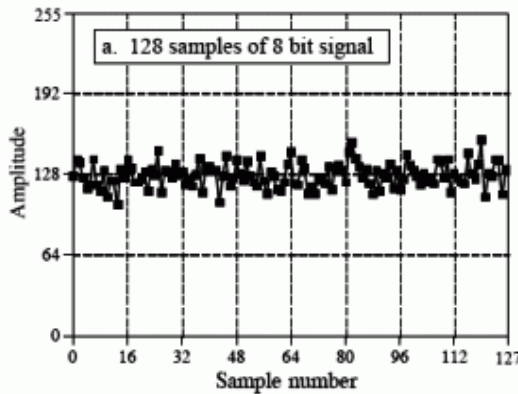


[4] Christopher M. Bishop, "Pattern Recognition and Machine Learning", Springer, 2006

[18] Kirkwood BR. Essentials of medical statistics. Oxford, Blackwell Science, 1988.

# 8. Basic Signal Statistics

## Histogram, Probability mass function, Probability density function



[4 ] Christopher M.Bishop, “ **Pattern Recognition and Machine Learning**”, Springer, 2006

[18] Kirkwood BR. Essentials of medical statistics. Oxford, Blackwell Science, 1988.

# 9. Problems in biomedical signal processing

- Accessibility of the variables to measurement
- Patient safety, preference for noninvasiveness
- Indirect measurements (variables of interest are not accessible)
- Variability of the signal source
- Interactions among physiological system
- Acquisition interference

[10] B H Brown, R H Smallwood, D C Barber, P V Lawford and D R Hose, “ **MEDICAL PHYSICS AND BIOMEDICAL ENGINEERING**”

[3] Alan V. Oppenheim, Ronald W. Schafer, John R. Buck, “ **Discrete-Time Signal Processing**”, Prentice-Hall, Inc. 1999, 1989



# 9.Problems in biomedical signal processing-Artefacts and interference

- Interference from other physiological systems (e.g. muscle artifacts in EEG recordings)
- Low-level signals (e.g. microvolts in EEG) require very sensitive amplifiers; they are easily sensitive to interference.
- Limited possibilities for shielding or other protection - Nonlinearity and obscurity of the system under study
- basically all biological systems exhibit nonlinearities while most of the methods are based on the assumption of linearity
- exact structures and true function of many physiological systems are often not known

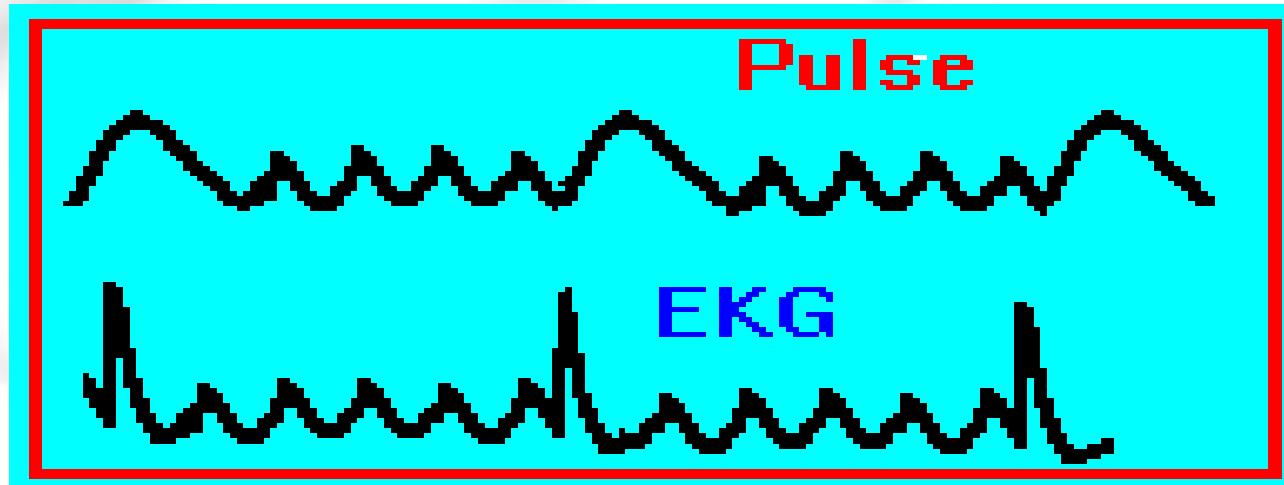
[3] Alan V.Oppenheim, Ronald W.Schafer, John R.Buck, “ **Discrete-Time Signal Processing**”, Prentice-Hall, Inc.1999,1989

[10] B H Brown, R H Smallwood, D C Barber, P V Lawford and D R Hose, “ **MEDICAL PHYSICS AND BIOMEDICAL ENGINEERING**”



# 9. Problems in biomedical signal processing-Artefacts and interference

## ➤ Some EEG artefacts



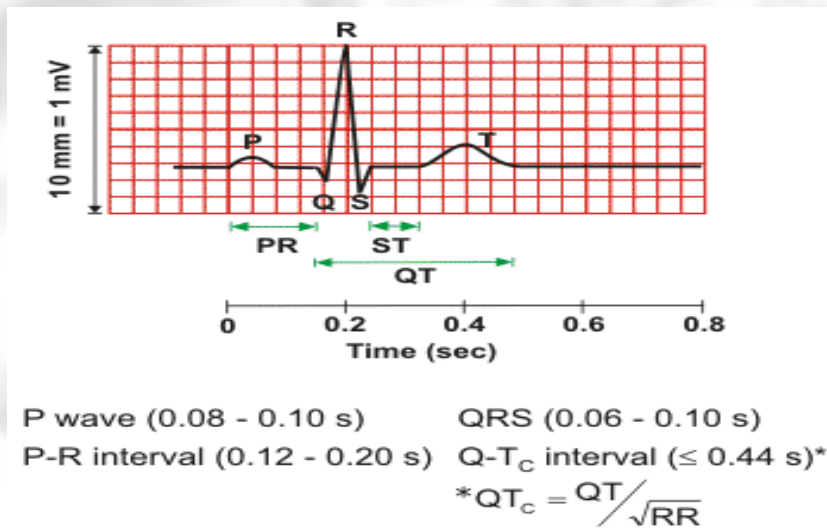
Pulse wave artefact: movement of electrode arising from patient pulse under the electrode.

ECG signal artefact: ECG signal also picked up by the EEG electrodes.

Both easily recognized because they are periodic.

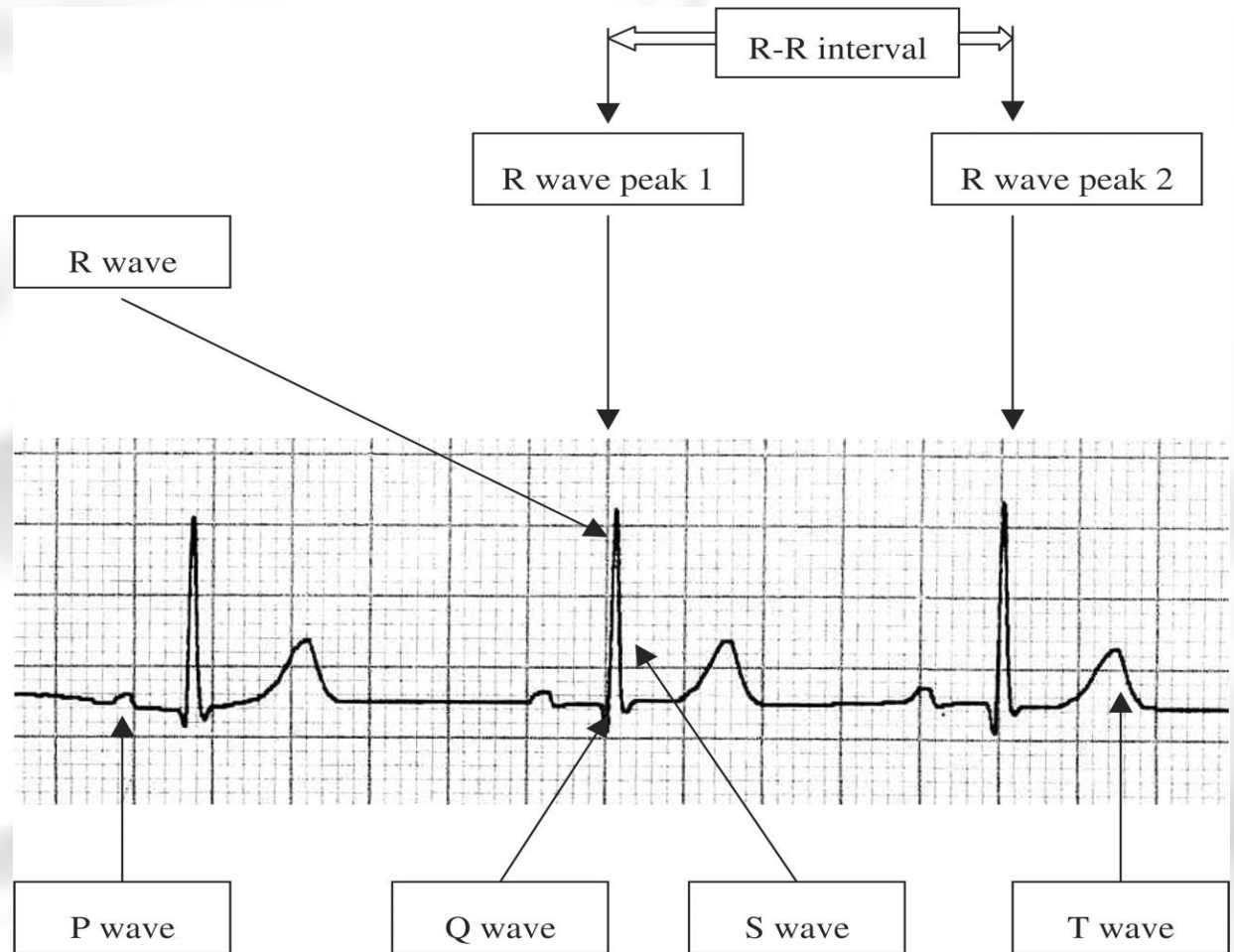
# 10.APPLICATION-ECG

- The electrocardiogram (ECG) is a time-varying signal reflecting the ionic current flow which causes the cardiac fibers to contract and subsequently relax. The surface ECG is obtained by recording the potential difference between two electrodes placed on the surface of the skin. A single normal cycle of the ECG represents the successive atrial depolarisation/repolarisation and ventricular depolarisation/repolarisation which occurs with every heart beat.
- Simply put, the ECG (EKG) is a device that measures and records the electrical activity of the heart from electrodes placed on the skin in specific locations
- A typical ECG period consists of P,Q,R,S,T and U waves



# 10.APPLICATION-ECG

The normal electrocardiogram with component waves labelled.



P wave: the sequential activation (depolarization) of the right and left atria  
QRS complexes: right and left ventricular depolarization  
T wave: ventricular repolarization  
U wave: origin not clear, probably "afterdepolarizations" in the ventricles

Reed M et al. QJM 2005;98:87-95

# 10.APPLICATION-ECG Filtering

- Three common noise sources
  - Baseline wander
  - Power line interference
  - Muscle noise
- When filtering any biomedical signal care should be taken not to alter the desired information in any way
- A major concern is how the QRS complex influences the output of the filter; to the filter they often pose a large unwanted impulse
- Possible distortion caused by the filter should be carefully quantified

# 10.APPLICATION-ECG Filtering

- Both baseline wander and powerline interference removal are mainly a question of filtering out a narrow band of lower-than-ECG frequency interference.
  - The main problems are the resulting artifacts and how to optimally remove the noise
- Muscle noise, on the other hand, is more difficult as it overlaps with actual ECG data
- For the varying noise types (baseline wander and muscle noise) an adaptive approach seems quite appropriate, if the detection can be done well. For power line interference, the nonlinear approach seems valid as ringing artifacts are almost unavoidable otherwise

[10] B H Brown, R H Smallwood, " **MEDICAL PHYSICS AND BIOMEDICAL ENGINEERING**", University of Sheffield, 1999

[13] Chaudhuri S., Pawar T.D., Duttagupta S., " **Ambulation Analysis in Wearable ECG** ", Springer, 2009

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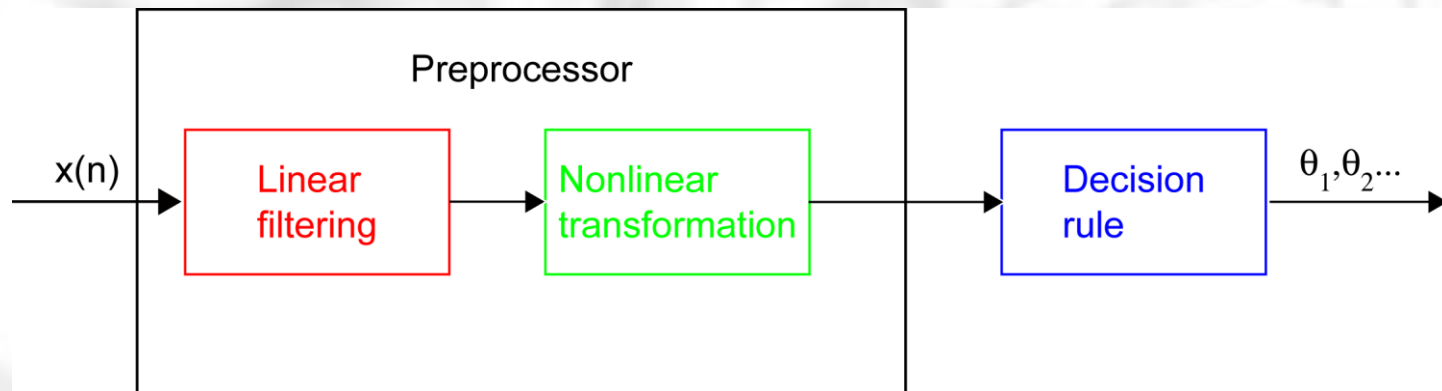
# 10.APPLICATION-QRS detection

- QRS detection is important in all kinds of ECG signal processing
- QRS detector must be able to detect a large number of different QRS morphologies
- QRS detector must not lock onto certain types of rhythms but treat next possible detection as if it could occur almost anywhere
- Typical structure of QRS detector algorithm: preprocessing (linear filter, nonlinear transformation) and decision rule
- For different purposes (e.g. stress testing or intensive care monitoring), different kinds of filtering, transformations and thresholding are needed
- Multi-lead QRS detectors



# 10.APPLICATION-QRS detection

- Bandpass characteristics to preserve essential spectral content (e.g. enhance QRS, suppress P and T wave), typical center frequency 10 - 25 Hz and bandwidth 5 - 10 Hz
- Enhance QRS complex from background noise, transform each QRS complex into single positive peak
- Test whether a QRS complex is present or not (e.g. a simple amplitude threshold)



# 10.APPLICATION-Estimation Problem

- Maximum likelihood (ML) estimation technique to derive detector structure
- Starting point: same signal model as for derivation of Woody method for alignment of evoked responses with varying latencies

$$x(n) = \begin{cases} v(n) & 0 \leq n \leq \theta-1 \\ s(n-\theta) + v(n) & \theta \leq n \leq \theta+D-1 \\ v(n) & \theta+D \leq n \leq N-1 \end{cases}$$

$x(n)$  observed signal

$s(n)$  QRS, known morphology

$v(n)$  noise

$\theta$  QRS occurrence time

$D$  duration of  $s(n)$

$N$  observation interval

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[13] Chaudhuri S., Pawar T.D., Duttagupta S., " **Ambulation Analysis in Wearable ECG** ", Springer, 2009

[14] Gari D. Clifford, Francisco Azuaje, Patrick E. McSharry, " **Advanced Methods and Tools for ECG Data Analysis** ", Artech House Publishers



# 10.APPLICATION-QRS detection

Unknown time of occurrence  $\theta$

ML estimate of occurrence time  $\theta$

(value that maximizes log likelihood function:)

$$\hat{\theta} = \arg \max_{\theta} \ln p(x; \theta)$$

PDF of observed signal

equivalent to finding peak amplitude in signal  $y(\theta)$

$$\hat{\theta} = \arg \max_{\theta} y(\theta)$$

filtering operation

# 10.APPLICATION-QRS detection

$y(\theta)$  is output of matched filter  $h(n)$

$$y(\theta) = \sum_{n=\theta}^{\theta+D-1} x(n)h(\theta - n)$$

False detection, because assumed one QRS complex present in N  
→ thresholding

$$\bar{y}(\hat{\theta}) > n$$

⇒ detected QRS complexes at  $\theta_1, \theta_2, \dots$

# 10.APPLICATION-QRS detection

Unknown time of occurrence and amplitude  $a$

observed signal  $x(n) = as(n - \theta) + v(n)$

maximize log-likelihood function

$$[\hat{\theta}, \hat{\alpha}] = \arg \max_{\theta} \ln p_v(x; \theta, a)$$

ML estimator of  $\theta$

$$\hat{\theta} = \arg \max_{\theta} \left[ \frac{E_s}{2\sigma_v^2} [\bar{y}^2(\theta)] \right]$$

thresholding

$$\bar{y}^2(\hat{\theta}) > n$$

# 10.APPLICATION-QRS detection

Unknown time of occurrence, amplitude and width

width parameter  $\ell$  in model of QRS waveform  $s(n, l)$

ML estimator of  $\theta$

$$\hat{\theta} = \arg \max_{\theta} \left( \frac{1}{2\sigma_v^2} \max_l \left[ E_s(l) \bar{y}^2(\theta, l) \right] \right)$$

Energy of  $s(n)$  a function of  $\ell$ , can not be omitted from estimation of  $\theta$

$$E_s(\theta, l) = \sum_{n=\theta}^{\theta+D-1} s^2(n - \theta, l)$$

[20] Veena Hegde, "Review of Data Analysis Methods for Denoising and Characterising ECG", November 2009

[21] Timo Bragge, Mika P.Tarvainen and Pasi A.Karjalainen, "High-Resolution QRS Detection Algorithm for Sparsely Sampled ECG Recordings", IEEE Trans Biomed Eng, August 2004

[22] J.Pan and W.Tompkins, "A real-time QRS detection algorithm", IEEE Trans Biomed ENG, vol.32, no.3, pp.230-236, March 1985

# 10.APPLICATION-QRS detection

Easier approach to model width:  $s(n)$  composed of two identical waveforms,  $q(n)$ , of which one is shifted  $l$  samples in time and with opposite sign

$$s(n, l) = q(n) - q(n - l)$$

$$\hat{\theta} = \arg \max_{\theta} \left( \frac{E_q}{\sigma_v^2} \max_l \left[ (1 - \rho_q(l)) (\bar{y}_q(\theta) - \bar{y}_q(\theta - l))^2 \right] \right)$$

maximized when  $\bar{y}_o(\theta)$  and  $\bar{y}_o(\theta - l)^2$   
positive maximum and negative minimum

→ Approximate, but computationally efficient ML estimator determines local extreme values of filtered signal for estimation of  $\theta$

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# 10.APPLICATION-QRS detection

Peak-and-valley picking strategy:

- Use of local extreme values as basis for QRS detection
- Base of several QRS detectors
- Distance between two extreme values must be within certain limits to qualify as a cardiac waveform
- Also used in data compression of ECG signals

# 10.APPLICATION-Linear Filtering

- To enhance QRS from background noise
- Examples of linear, time-invariant filters for QRS detection:
  - Filter that emphasizes segments of signal containing rapid transients (i.e. QRS complexes)
    - Only suitable for resting ECG and good SNR
  - Filter that emphasizes rapid transients + low pass filter
  - Family of filters, which allow large variability in signal and noise properties

$$H(z) = (1 - z^{-L_1})(1 + z^{-1})^{L_2}$$

difference between input  
and delayed input

lowpass filter

- Suitable for long-term ECG recordings (because no multipliers)
- Filter matched to a certain waveform not possible in practice
  - ⇒ Optimize linear filter parameters (e.g.  $L_1$  and  $L_2$ )



# 10.APPLICATION-Decision Rule

- To determine whether or not a QRS complex has occurred
- Fixed threshold  $\eta$
- Adaptive threshold
  - QRS amplitude and morphology may change drastically during a course of just a few seconds
- Here only amplitude-related decision rules
- Noise measurements
- Interval-dependent QRS detection threshold
  - Threshold updated once for every new detection and is then held fixed during following interval until threshold is exceeded and a new detection is found
- Time-dependent QRS detection threshold



# 10.APPLICATION-Performance Evaluation

- Before a QRS detector can be implemented in a clinical setup
  - Determine suitable parameter values
  - Evaluate the performance for the set of chosen parameters
- Performance evaluation
  - Calculated theoretically or
  - Estimated from database of ECG recordings containing large variety of QRS morphologies and noise types

# 10.APPLICATION-Performance Evaluation

- Estimate performance from ECG recordings database

$P_D$ : probability of true detection

$P_F$ : probability of false detection

$P_M$ : probability of missed detection

$N_D$ : number of correctly detected complexes

$N_F$ : number of false alarms

$N_M$ : number of missed beats

$\theta_j$ : estimated occurrence time

$\theta_i$ : annotation time

$\Delta\theta$ : matching window

$$\hat{P}_D = \frac{N_D}{N_D + N_M}$$

$$\hat{P}_F = \frac{N_F}{N_D + N_F}$$

A beat detected when

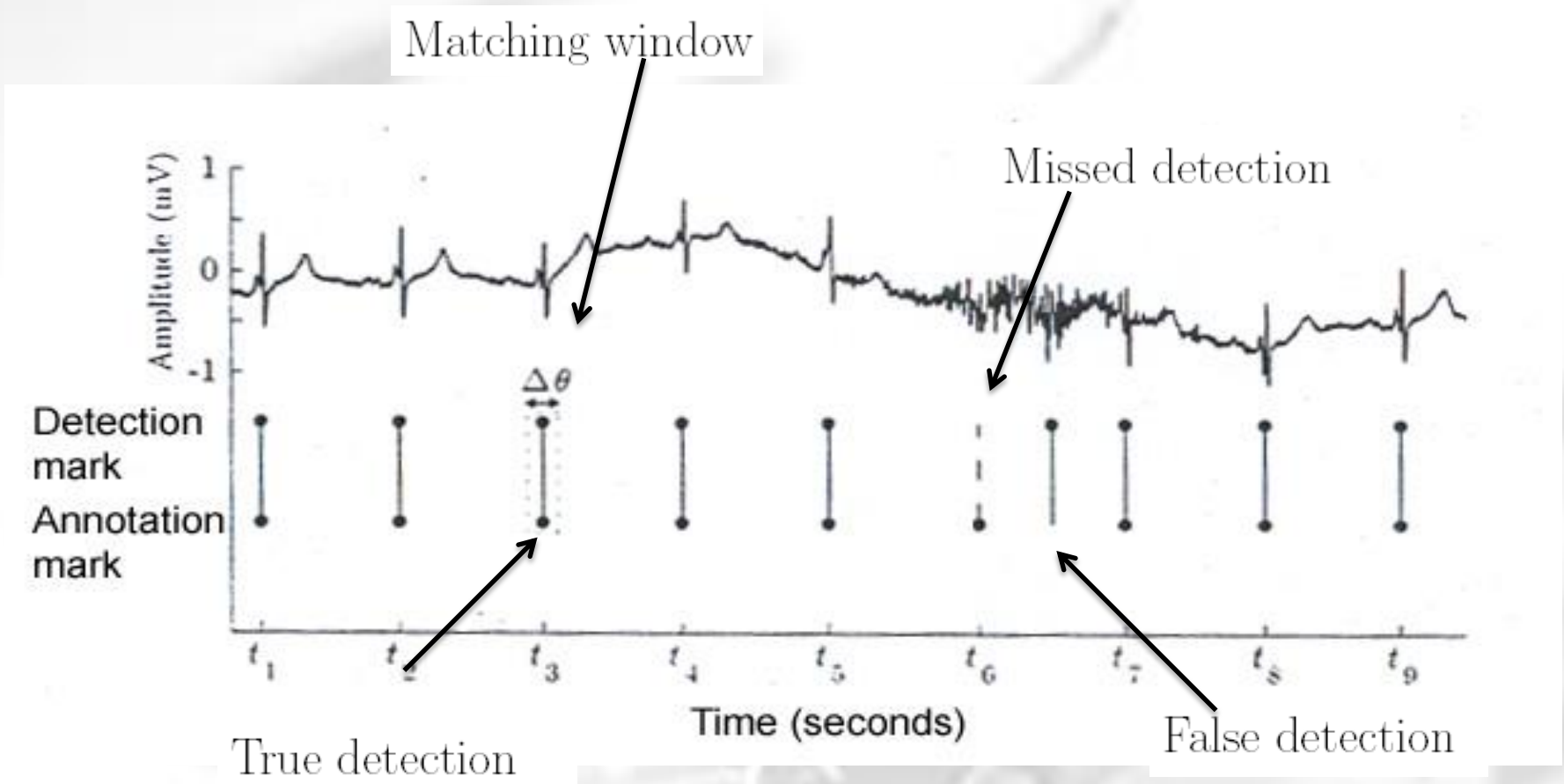
$$\left| \hat{\theta}_j - \theta_i \right| \leq \Delta\theta$$

[20] Veena Hegde, "Review of Data Analysis Methods for Denoising and Characterising ECG", November 2009

[21] Timo Bragge, Mika P.Tarvainen and Pasi A.Karjalainen, "High-Resolution QRS Detection Algorithm for Sparsely Sampled ECG Recordings", IEEE Trans Biomed Eng, August 2004

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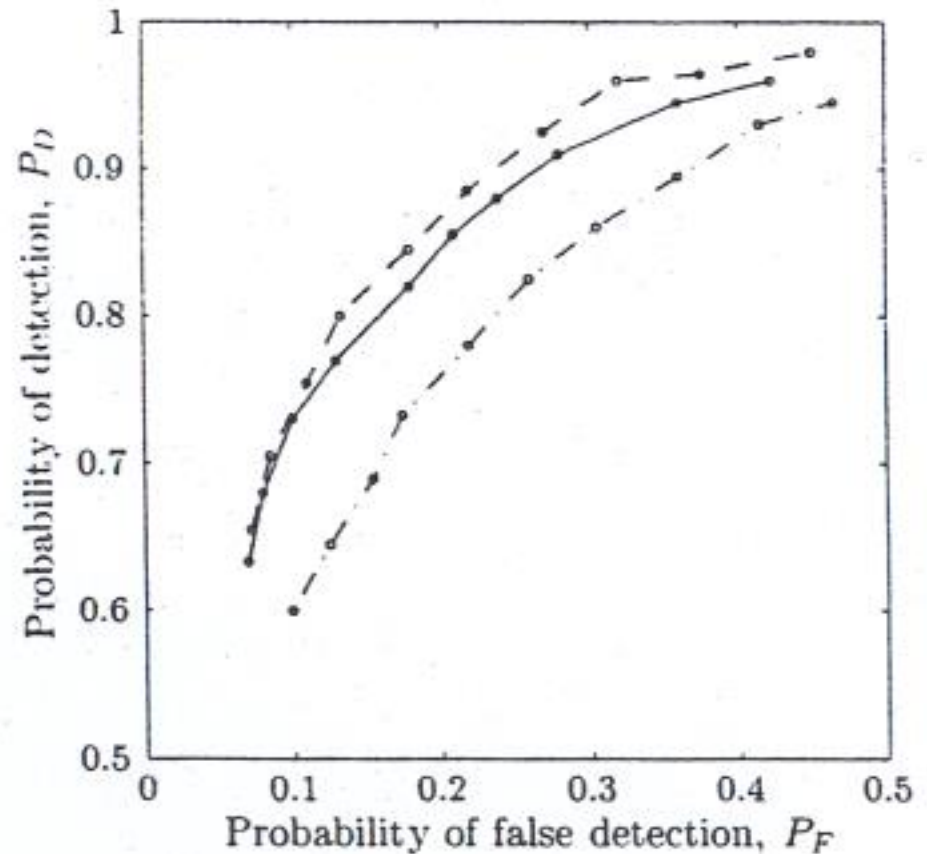
# 10.APPLICATION-Performance Evaluation



# 10.APPLICATIONS-ECG- Performance Evaluation

## Receiver operating characteristics (ROC)

- Study behaviour of detector for different parameter values
- Choose parameter with acceptable trade-off between  $P_D$  and  $P_F$



[20] Veena Hegde, "Review of Data Analysis Methods for Denoising and Characterising ECG", November 2009

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# 11.PROBLEMS-Short questions

1. Would you consider a nerve action potential as a continuous or discontinuous signal?
2. Is the ECG a periodic signal?
3. What is the result of carrying out a Fourier transform on a rectangular impulse in time?
4. Is the variance of a data set equal to the square root of the standard deviation?
5. Is an EMG signal periodic?
6. What is the convolution integral?
7. What do you get if you multiply the Fourier transform of a signal by the frequency response of a system?
8. Measurements are made on a group of subjects during a period of sleep. It is found that the probability of measuring a heart rate of less than 50 bpm is 0.03. In the same subjects a pulse oximeter is used to measure oxygen saturation  $PO_2$  and it is found that the probability of measuring a value of  $PO_2$  below 83% is 0.04. If the two measurements are statistically independent then what should be the probability of finding both a low heart rate and low oxygen saturation at the same time?  
If you actually find the probability of both the low heart rate and low oxygen saturation occurring at the same time to be 0.025 then what conclusion would you draw?

# 11.PROBLEMS-Answers

1. A nerve action potential should probably be considered as discontinuous as it moves very rapidly between the two states of polarization and depolarization.
2. The ECG is periodic, although the R-R interval is not strictly constant.
3. You obtain a frequency spectrum of the form  $\sin(t)/t$  if you carry out a Fourier transform on a rectangular impulse.
4. No, the variance is equal to the square of the standard deviation.
5. An EMG signal is not periodic. It is the summation of many muscle action potentials which are asynchronous.
6. The convolution integral gives the output of a system in terms of the input and the characteristic response of the system to a unit impulse.
7. If you multiply the FT of a signal by the frequency response of a system then you get the FT of the output from the system.
8. The combined probability if the two measurements are independent would be 0.0012. If the probability found was 0.025 then the conclusion would be that heart rate and oxygen saturation measurements are not statistically independent. This would not be a surprising finding as the two measurements have a physiological link.



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